An investigation of Hardy's Paradox using weak measurements

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CLEO/QELS May 2005

Funding by:

NSERC CRSNG
ICIP CIPI
DARPA
PHOTONICS RESEARCH ONTARIO

Motivation

Hardy’s Paradox

Direct contradiction involving the results of two observers
- logical inequality

David Mermin: Hardy's Paradox “stands in its pristine simplicity as one of the strangest and most beautiful gems yet to be found in the extraordinary soil of quantum mechanics” - N. D. Mermin, Am. J. Phys. 62, 880 (1994).

Problem: The measurements leading to Hardy’s Paradox do not commute. Since they disturb the system we can not perform them simultaneously to test their veracity.

Solution: Eliminate (or at least minimize) the disturbance

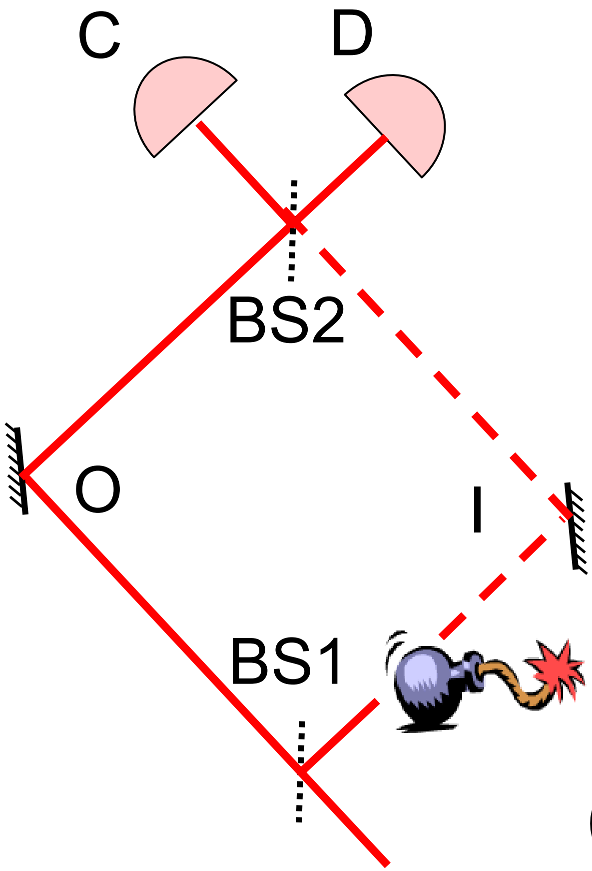
How: Turn down the coupling to the measurement device

Weak Measurement

Warning:
Controversial!

Interaction-Free Measurement

A. C. Elitzur, and L. Vaidman, Found. Phys. **23**, 987 (1993)



Bomb Absent:

Only detector C fires

Bomb Present:

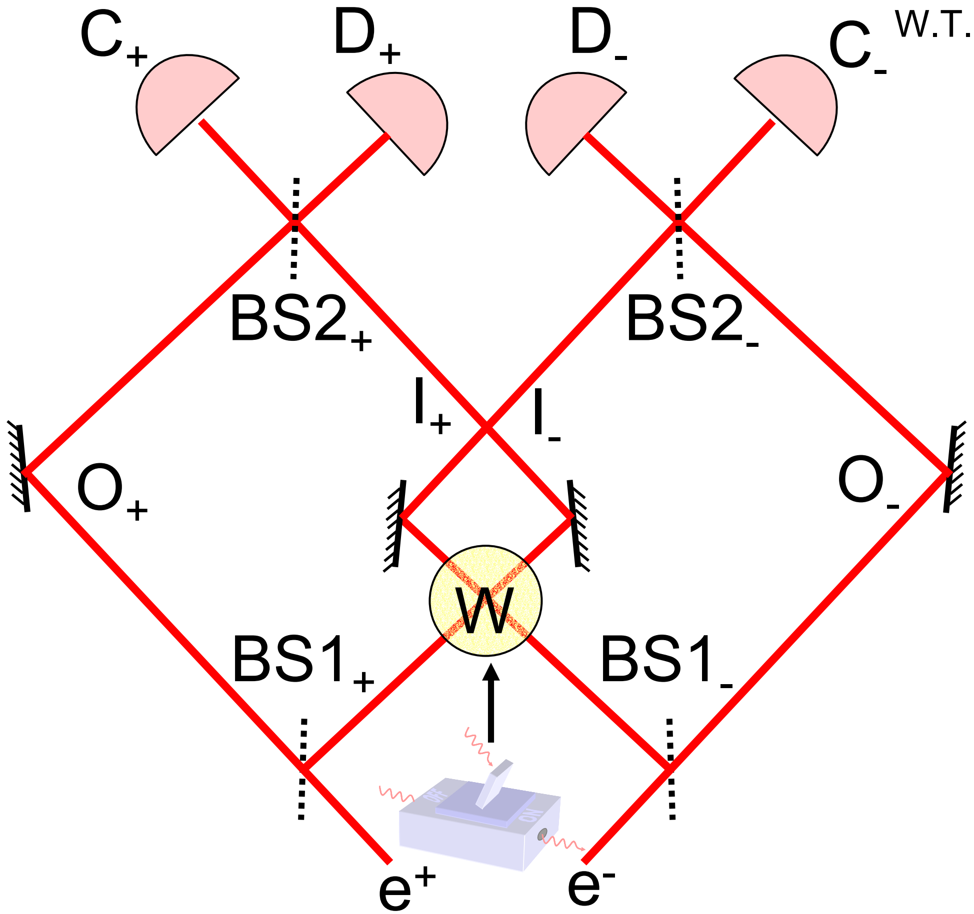
Detector	**Prob.**	**Result**
C	¼	None
Neither	½	Bang
D	¼	Present

Interaction-Free Measurement: The bomb is detected without detonating it.

- Indirect measurement
- Still works if bomb is in a quantum superposition

Hardy's Paradox

L. Hardy, Phys. Rev. Lett. **68**, 2981 (1992)
W.T.M. Irvine, et al. quant-ph/0410160



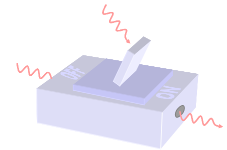
Observations:
Whenever $D_+ \rightarrow I_-$
Whenever $D_- \rightarrow I_+$
Sometimes D_+ & D_-

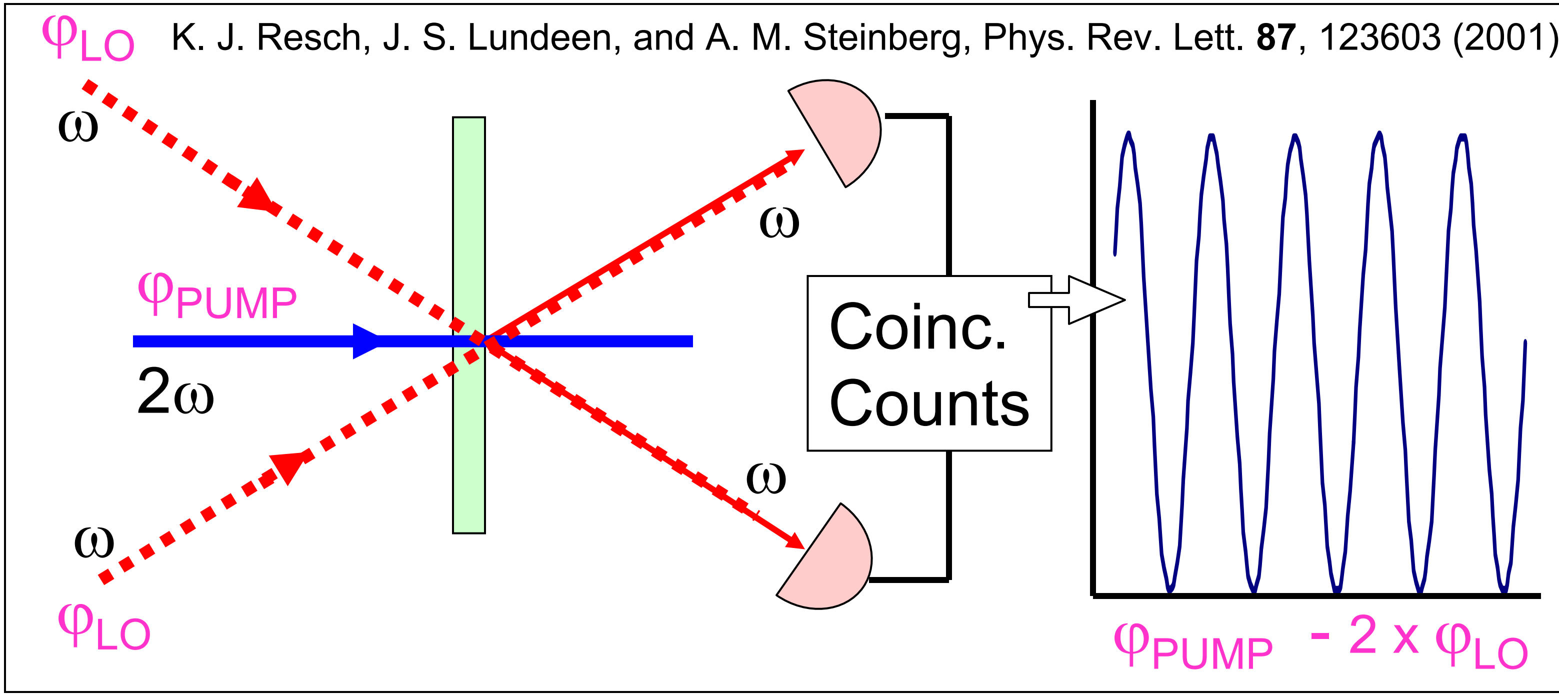
Logical Implication:
Sometimes I_- & I_+

Paradox:
We never find the particles in I_- & I_+

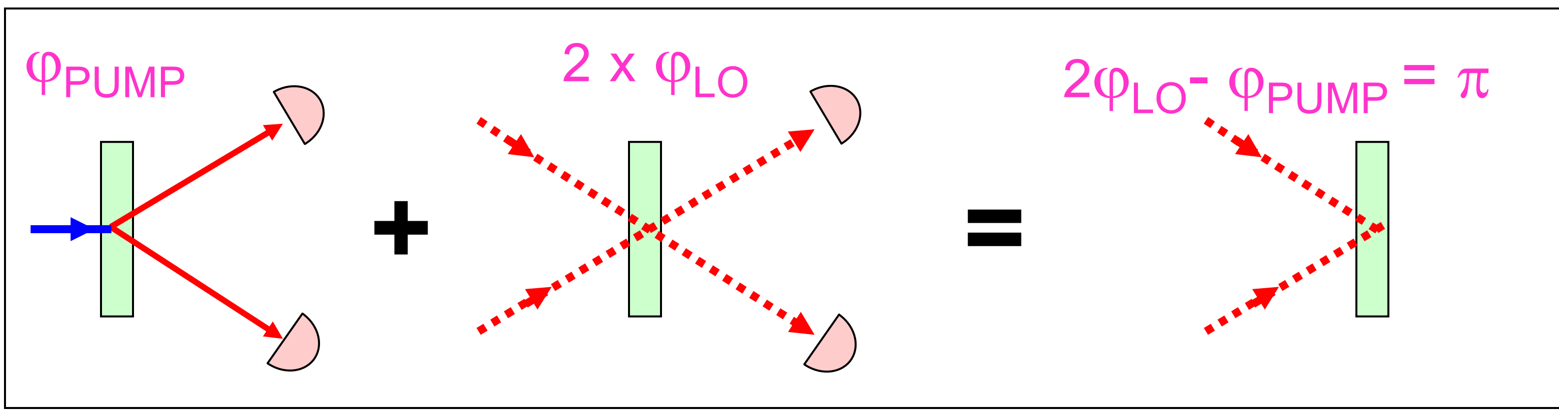
- Can we talk about the past in postselected QM?
- How should we interpret indirect quantum measurements?

The Switch



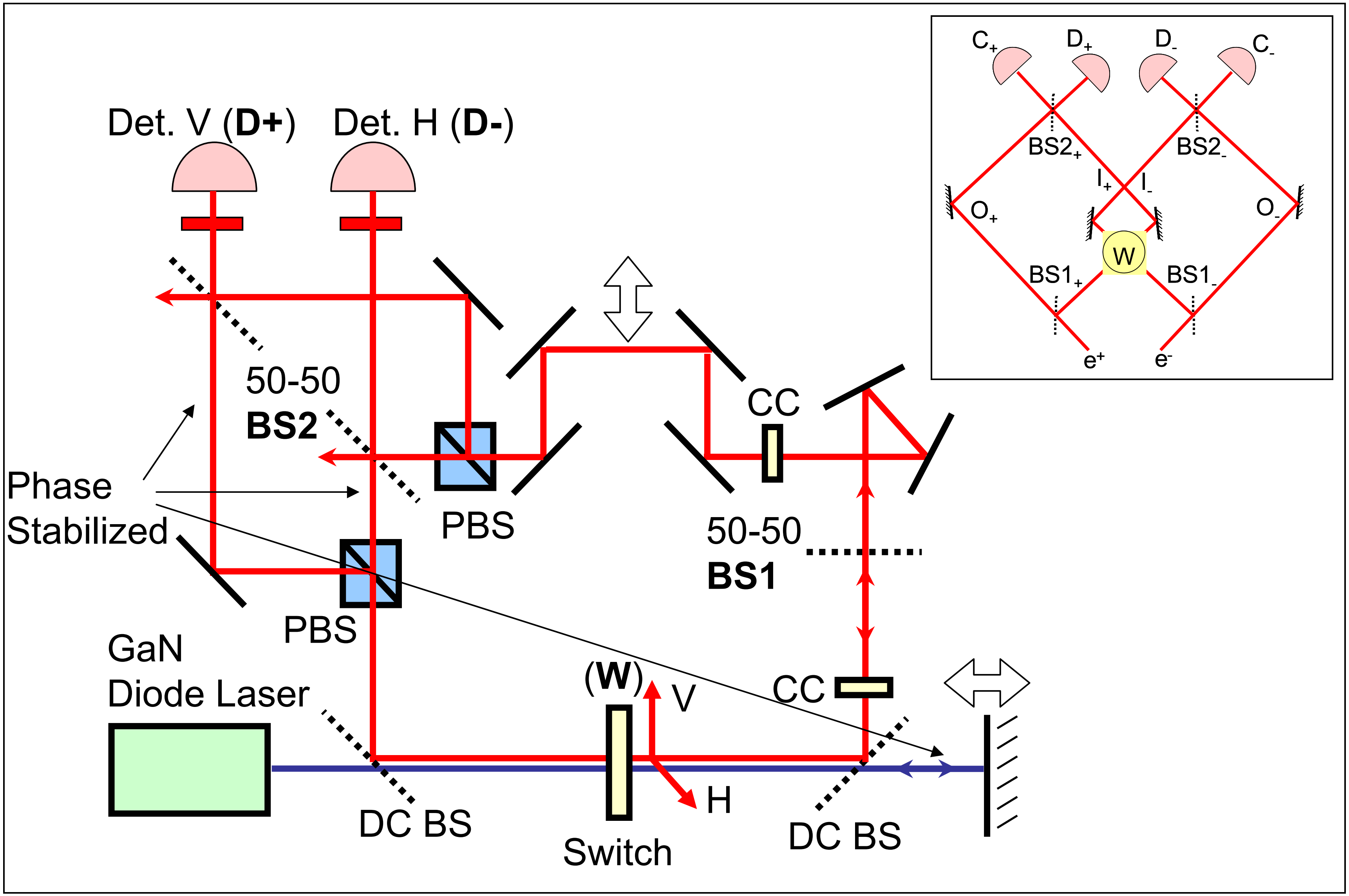


φLO
K. J. Resch, J. S. Lundeen, and A. M. Steinberg, Phys. Rev. Lett. **87**, 123603 (2001).
ω
ω
φPUMP
2ω
Coinc.
Counts
ω
ω
φLO
φPUMP - 2 x φLO

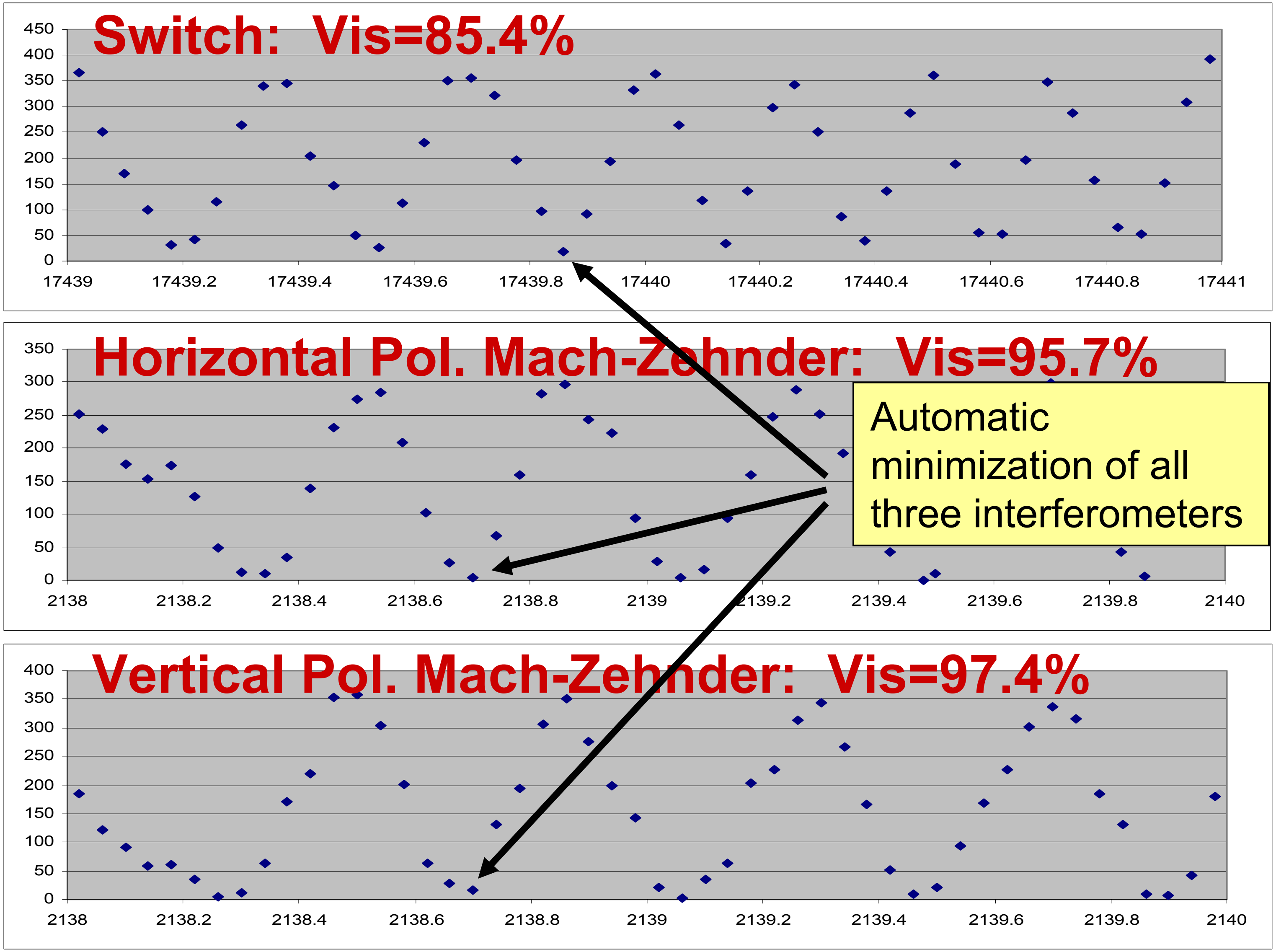


φPUMP
+
2 x φLO
=
2φLO- φPUMP = π

Experimental Setup



Experimental Data



Experimental Data

Testing IFM+

If D+ clicks ⇒

Photon is in arm I-	96%
Photon is in arm O-	4%

Testing IFM-

If D- clicks ⇒

Photon is in arm I+	97%
Photon is in arm O+	3%

Testing Switch

Rate of photon pairs in I+ and I-
= 10.4 ± 0.33/5s

The Paradox

Rate of D+ and D- coincidences
= 7.28 ± 0.41/5s

Weak Measurements

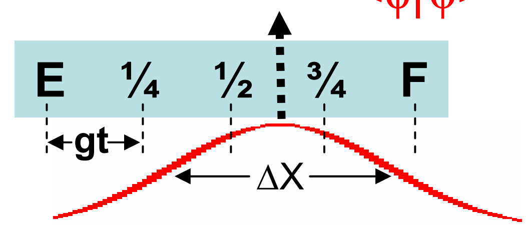
Aharonov, Albert,&Vaidman , PRL 60, 1351 ('88)

Measurement of $\hat{A}$	**Pointer Position Uncertainty**	**$H_{int}=gP\hat{A}$**
Ideal	Dirac Delta	$\Delta X=0$
Real	Width << Change in Position	$\Delta X \ll gt$
Weak	Width >> Change in Position	$\Delta X \gg gt$

$\text{Pointer}(X)=\exp[-(X-gtA_W)^2/\Delta X]$

Average shift of pointer:

Weak Value = $A_W=\dfrac{\langle\phi|A|\psi\rangle}{\langle\phi|\psi\rangle}$



Since: $\Delta \mathbf{X} \Delta \mathbf{P} \geq \mathbf{h/2}\pi$

$\Rightarrow$ small disturbance

$\Rightarrow$ little system – pointer entanglement

Useful for investigating post-selected systems: **Hardy's Paradox**

For the paradoxical result (Post-selecting on D_+ & D_- click):

Weakly measure which arms the particles were in, individually and as **pairs**.

Two-Particle Weak Measurements

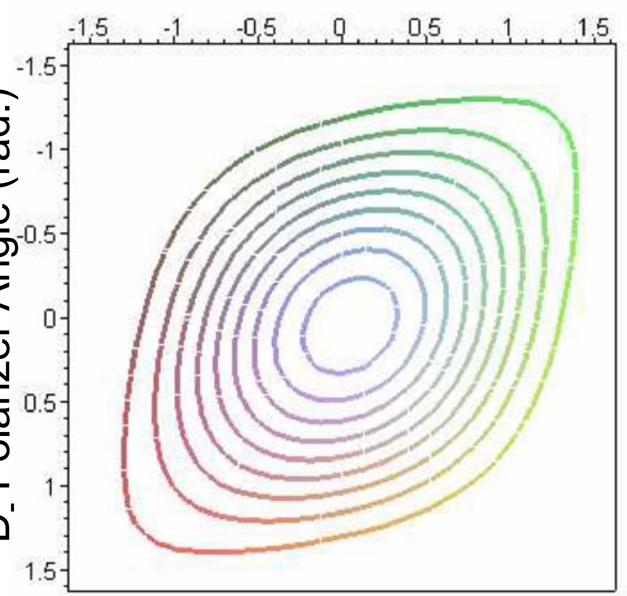
• Problem: For two-particle weak measurements we need a strong nonlinearity to implement a Von Neuman measurement interaction ($H_{int}=gP\hat{A}_1\hat{A}_2$).

• Solution: Do two single particle weak measurements → Measure correlations in the two separate pointers

Pointer Polarization Correlations for $\langle \hat{A}_1\hat{A}_2 \rangle_{weak}$

D$_+$ Polarizer Angle (rad.)

D$_-$ Polarizer Angle (rad.)



Weak Measurement for a Polarization Pointer (N particles):

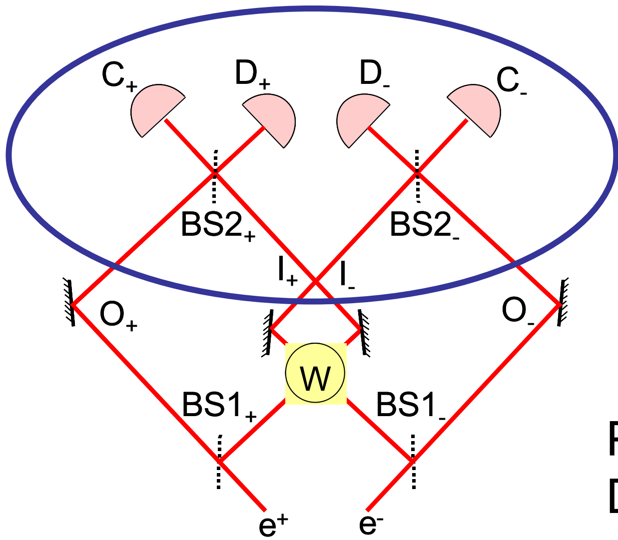
$$\left\langle \prod_{j=1}^{N} \hat{A}_j \right\rangle_W = \left\langle \prod_{j=1}^{N} \hat{S}_{jz}^{-} \right\rangle_{fi} \left(\frac{1}{gt\hbar s} \right)^N$$

Spin Lowering Operator

Lundeen & Resch, Phys. Lett. A 334 (2005) 337–344
Resch & Steinberg, PRL 92,130402 (2004)

Weak Measurements in Hardy's Paradox

Y. Aharanov, A. Botero, S. Popescu, B. Reznik, J. Tollaksen, Phys. Lett. A 301, 130 (2001)



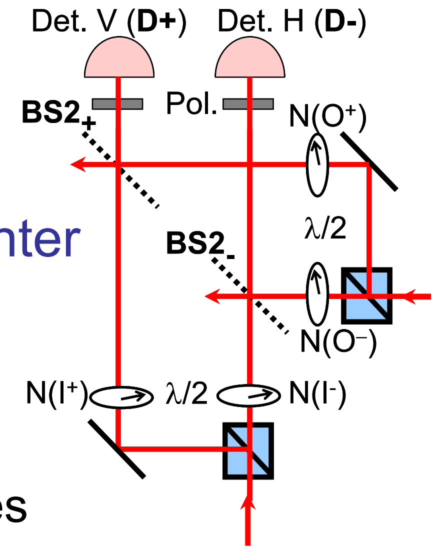
Polarization Pointer

$H_{int}=gS_Y\hat{A}$

$\Delta\theta=20°$

Post-selecting on

D_+ & D_- coincidences



Truth Table for Weak Values (Probabilities) of arm occupation: $|N\rangle\langle N|$

Occupation Probability	$N(I^-)=1$ **0.84 ± 0.01**	$N(O^-)=0$ **-0.08 ± 0.01**
$N(I^+)=1$ **0.98 ± 0.01**	0 **0.25 ± 0.02**	1 **0.60 ± 0.02**
$N(O^+)=0$ **0.13 ± 0.01**	1 **0.67 ± 0.02**	-1 **-0.73 ± 0.02**

Theory (ideal) **Results**

-'ve value resolves paradox!

Conclusions

- A single-photon level switch allows for the implementation of Hardy's Paradox.

- Weakly measuring where in the interferometers the photons were gives results that resolve the paradox.

- This is the first experimental two-particle weak measurement.

- Weak measurements are useful for investigating post-selected systems (e.g. LOQC)

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