Experimental Weak Measurements in Hardy's Paradox

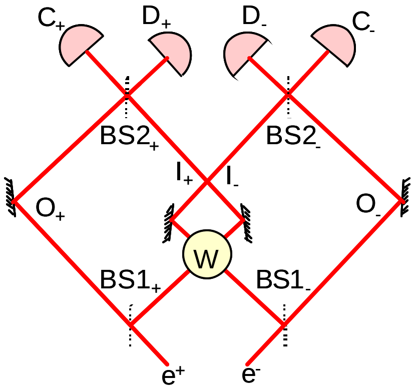
Jeffrey S. Lundeen, and Aephraim M. Steinberg
Department of Physics, University of Toronto, 60 St.George Street, Toronto, Ontario M5S 1A7 CANADA
Ph: 416-946-3162, Fax: 416-978-2537, lundeen@physics.utoronto.ca

Abstract: Hardy's Paradox brings to light some of the striking difficulties in the interpretation of indirect quantum measurements. We weakly measure the location of the particles in Hardy's Paradox. The results provide a self-consistent resolution.

OCIS codes: (270.5290) Quantum Optics, (000.1600) Classical and quantum physics

1. Introduction

Hardy's Paradox was introduced in 1992 as a particularly clear example of a contradiction between classical reasoning and the predictions of quantum mechanics [1]. Later, by changing from particles in different paths to polarized particles, Hardy generalized his scheme and maximized the number of events in which the contradiction occurs [2]. The original scheme remains the simplest to understand and was later studied by Aharonov in the context of weak measurements [3]. Since weak measurements disturb a quantum system minimally, they can be used to study the properties of post-selected systems before the post-selection. In the case of Hardy's Paradox, the post-selected ensemble corresponds to the detection events that lead to the contradiction. Aharonov found that the weak measurement results, known as weak values, give us a strange but self-consistent resolution to the paradox. We experimentally implement Hardy's Paradox with a new photon-photon switch [4] and perform the weak measurements.



Outcome	Prob
D_+ and C_-	1/16
D_- and C_+	1/16
C_+ and C_-	9/16
D_+ and D_-	1/16
Explosion	4/16

Fig. 1. The original form of Hardy's Paradox is two nested Interaction-Free Measurements. The electron is the object for the positron interferometer and vice versa. The bright and dark ports are labeled C and D respectively. A coincident detection at D_+ and D_- implies both particles were in region W and should have annihilated each other.

2. Hardy's Paradox

Hardy's paradox results from applying classical reasoning to a quantum system, namely a pair of Mach-Zehnder interferometers (MZI), each containing one particle. Initially, each MZI is aligned so that the particle always leaves through one exit port, termed the "bright" port. If a perfectly absorbing object blocks one of the two arms of the interferometer, interference is disrupted and there is now a finite possibility to detect the particle at the "dark" port of the interferometer. Since this detection indicates the presence of the object and also that the particle was not absorbed by the object, this is called an Interaction-Free Measurement [5]. In Hardy's Paradox, one arm from each interferometer (the "Inner" arm) crosses through a common point, W. It is postulated that if the both particles cross point W simultaneously they will interact and annihilate each other. In other words, each particle acts as an object for the other particle so that each MZI performs an Interaction-Free Measurement on the particle in the other interferometer. In this setup, quantum mechanics predicts that occasionally one will simultaneously detect the particles at the two "dark" exit ports. This implies that both particles were simultaneously at point W and should have annihilated. The paradox is that the particles were detected and, hence, did not annihilate. This is generally interpreted as indicative of the dangers of counterfactual reasoning (reasoning about measurements which were not performed); according to Copenhagen, one cannot talk about which path one of these particles took within the interferometer, despite the apparent, yet indirect, evidence provided by the dark-port detection.

Previously, it was difficult to ensure that two particles suitable for interferometry would annihilate each other with high probability. Photons would be ideal particles to use but they interact very weakly, even in nonlinear crystals. In our laboratory, we have developed a single-photon level switch that, ideally, allows a photon pair passing through it to up-convert or “annihilate” with a probability of close to 100% [4]. This device consists of a nonlinear $\chi^{(2)}$ crystal with a strong pump beam passing through it. If phase matched properly, the two input modes of the switch will interfere with the fields created in the crystal to eliminate all photon pairs in the output modes. The two particles used are a horizontally polarized photon and a vertically polarized photon. This allows for a relatively simple setup where the two MZIs lie directly on top of each other. Recently, an alternate set up has been demonstrated [6].

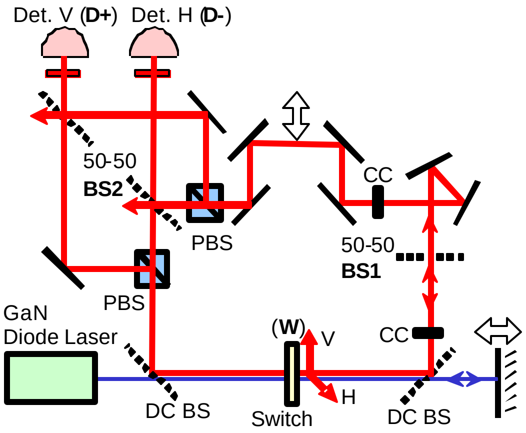


Fig. 2. The experimental setup. BS1 and BS2 correspond to the same named beamsplitters in figure 1. Dichroic beamsplitters are labeled DC BS and CC indicates a compensating crystal for birefringence correction. The two Mach-Zehnder Interferometers contain orthogonal polarizations of light and lie directly on top of each other until they are separated at the Polarizing Beam Splitters (PBS).

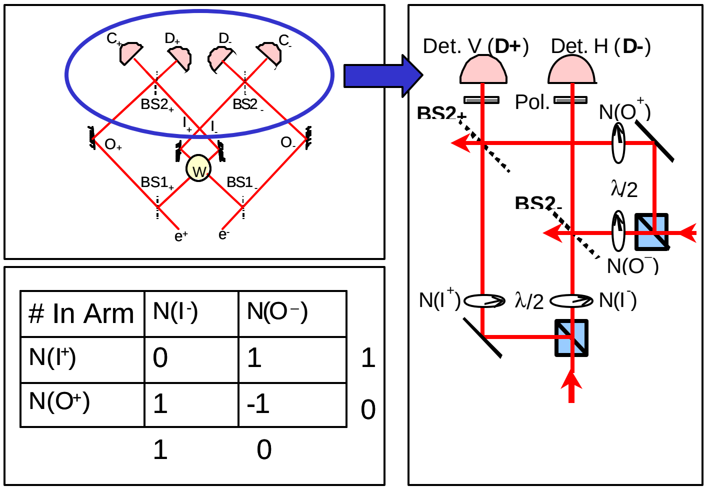
3. Weak Measurements

Weak measurement was originally proposed by Aharanov, Albert and Vaidman (AAV) [7] as an extension to the standard von Neumann (“strong”) model of measurement. A weak measurement can be performed by sufficiently reducing the coupling between the measuring device and the measured system. In this case, the pointer of the measuring device begins in a state with enough position uncertainty that any shift induced by the weak coupling is insufficient to distinguish between the eigenvalues of the observable in a single trial. While at first glance it may seem strange to desire a measurement technique that gives less information than the standard one, recall that the entanglement generated between the quantum system and measurement pointer is responsible for collapse of the wavefunction. Furthermore, if multiple trials are performed on an identically prepared ensemble of systems one can measure the average shift of the pointer to any precision - this average shift is called the weak value. A surprising characteristic of weak values is that they need not lie within the eigenvalue spectrum of the observable and can even be complex. On the other hand, an advantage of weak measurements is that they do not disturb the measured system nor any other simultaneous weak measurements or subsequent strong measurements, even in the case of non-commuting observables. This makes weak measurements ideal for examining the properties and evolution of systems before post-selection.

The IFM dark port detections only allow us to make counterfactual claims about which MZI arms the photons were in. Any strong measurements designed to verify these claims would disturb at least one of the MZIs, so that it would no longer act as an IFM. Thus the paradox would be destroyed. On the other hand, because they do not disturb the system, weak measurements can test these claims. Aharonov used weak measurement to find which Mach-Zehnder arms the photons were in, individually and as a pair, in the subensemble of systems for which the IFMs give their paradoxical result [3]. The resulting weak values for the single-photon occupation numbers (eg.

$|O_+\rangle\langle O_+|$) were found to be $N_{O+} = 0$, $N_{O-} = 0$, $N_{I+} = 1$, $N_{I-} = 1$, where I is the inner arm and O is the outer arm. These simply agree with the results of the IFMs: Individually, the photons were in the inner arms. The testable absence of photon pairs in the inner arms sets the photon pair weak value $N_{I,I}$ equal to zero. From this we find that $N_{O,I}$ and $N_{I,O}$ equal one since we already know $1 = N_{I+} = N_{I,I} + N_{I,O}$ for instance. But this leaves us with two photon pairs in total. To correct for this the last weak value $N_{O,O}$ is forced to be negative one. Although there has been a proposal to test Aharonov's strange prediction in an ion system [8], as of yet there have been no experiments.

Instead of coupling the particle observable to an external system, which would require multiparticle interactions, typically an internal degree of freedom of particle is used as the measurement pointer. In our case we rotate the polarization of the photon in the desired arm by only 10 degrees, small enough to minimally disturb the system. Then, to find the corresponding weak value, we measure the polarization rotation at the dark port of the interferometer. This procedure is sufficient for single-photon weak values but a similar one for the photon pair weak values would require multiparticle interactions as we would need to couple both photons to a single pointer. Recently, Resch and Steinberg devised a way of extracting two-particle weak values from correlations in the pointers of two individual weak measurements [9]. This was later extended to multiparticle weak values and greatly simplified as well [10,11]. We use this technique to weakly measure the photon pair occupation numbers.



# In Arm	$N(I^-)$	$N(O^-)$	
$N(I^+)$	0	1	1
$N(O^+)$	1	-1	0
	1	0	

Fig. 1. Half-waveplates are used to weakly rotate the polarization in the interferometer arm under study. Polarizers in front of the dark port detectors measure the subsequent rotation, which is proportional to the weak value. The logic table gives a summary of the theoretical weak values and shows their self-consistency.

4. Conclusion

This experiment is the first demonstration of the measurement of a multiparticle weak value. This type of weak value is particularly useful for investigating post-selected processes such as Hardy's Paradox. We set up Hardy's Paradox and weakly measure where the photons are, individually and as a pair. We expect this weak measurement technique will be useful for investigating applied post-selected multiparticle systems including quantum logic gates and the preparation of novel quantum states.

[1] L. Hardy, "Quantum mechanics, local realistic theories, and Lorentz-invariant realistic theories," Phys. Rev. Lett. **68**, 2981-2984 (1992).
[2] L. Hardy, "Nonlocality for two particles without inequalities for almost all entangled states," Phys. Rev. Lett. **71**, 1665-1668 (1993).
[3] Y. Aharonov, A. Botero, S. Popescu, B. Reznik, J. Tollaksen, "Revisiting Hardy's paradox: counterfactual statements, real measurements, entanglement and weak values," Phys. Lett. A **301**, 130-138 (2002).
[4] K. J. Resch, J. S. Lundeen, and A. M. Steinberg, "Nonlinear Optics with Less Than One Photon," Phys. Rev. Lett. **87**, 123603 (2001).
[5] A. Elitzur and L. Vaidman, "Quantum mechanical interaction-free measurements," Found. Phys. **23**, 987-997 (1993).
[6] W.T.M. Irvine, J.F. Hodelin, C. Simon, D. Bowmeester, "Realisation of Hardy's Thought Experiment," quant-ph/0410160 (2004).
[7] Y. Aharonov, D. Z. Albert, and L. Vaidman, "How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100," Phys. Rev. Lett. **60**, 1351-1354 (1988).
[8] K. Mølmer, "Counterfactual statements and weak measurements: an experimental proposal," Phys. Lett. A **292**, 151-155" (2001).
[9] K. J. Resch and A. M. Steinberg, "Extracting Joint Weak Values with Local, Single-Particle Measurements," Phys. Rev. Lett. **92**, 130402 (2004).
[10] K.J. Resch, "Practical weak measurement of multiparticle observables," J. Opt. B **6**, 482-487 (2004).
[11] J. S. Lundeen and K. J. Resch, accepted to Phys. Lett. A (2004).