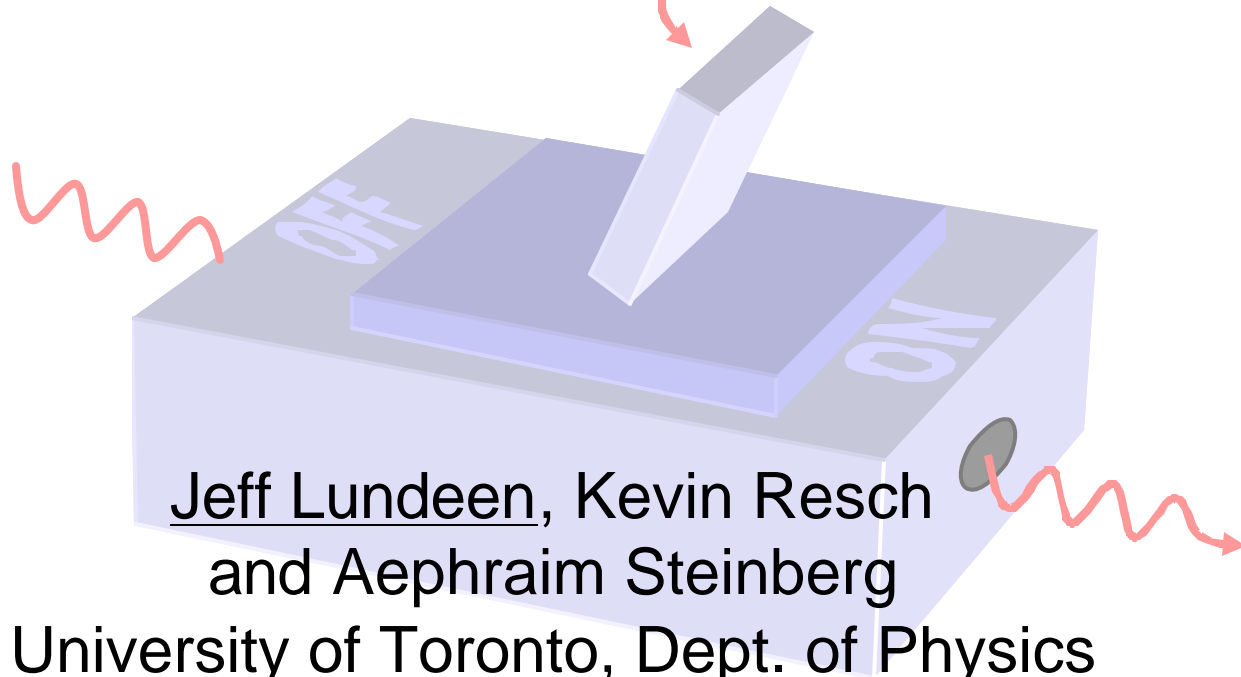


Applications of a nonlinear photon switch to Hardy's Paradox and Bell-state determination



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PQE XXXV

Financial Support from NSERC, CFI, and Photonics
Research Ontario, DARPA QuIST



Can we construct a two-photon gate?

Photons do not naturally interact: Great for transmission. Not so great for calculation.

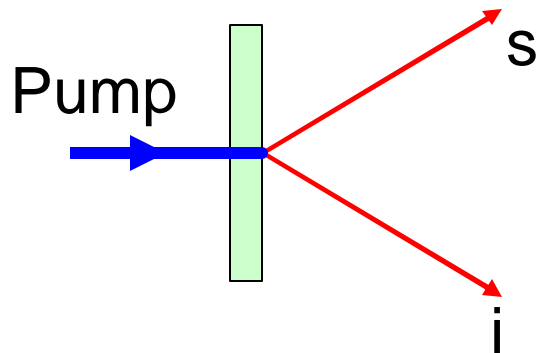
Proposed Solutions:

- **Better materials by a factor of 10^{10}**
Absorptive nonlinearities (Franson), Resonance in Micro-structures (Gaeta, Walmsley)
- **Cavity Quantum Electrodynamics**
Haroche, Kimble, Walther, Rempe
- **EIT**
Harris, Scully, Lukin, Fleishhauer, Hau
- **Measurement-induced nonlinearities**
Knill, Laflamme, Milburn, Franson, White, Zeilinger
- **Interference-enhanced nonlinearities**
Exchange effects in atomic clouds (Franson), $\chi^{(2)}$ with interference (Steinberg)

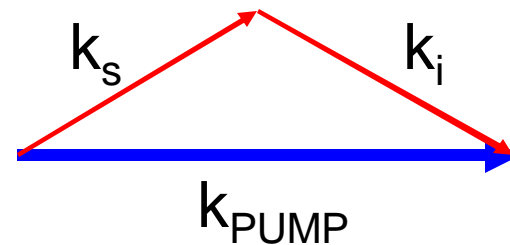


Spontaneous Parametric Downconversion

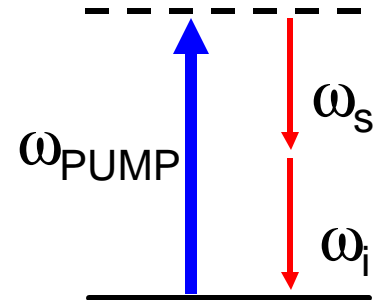
Downconversion



Momentum is conserved..



..as well as energy

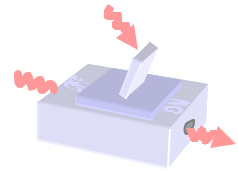


$$\varphi_{\text{PUMP}} = \varphi_s + \varphi_i$$

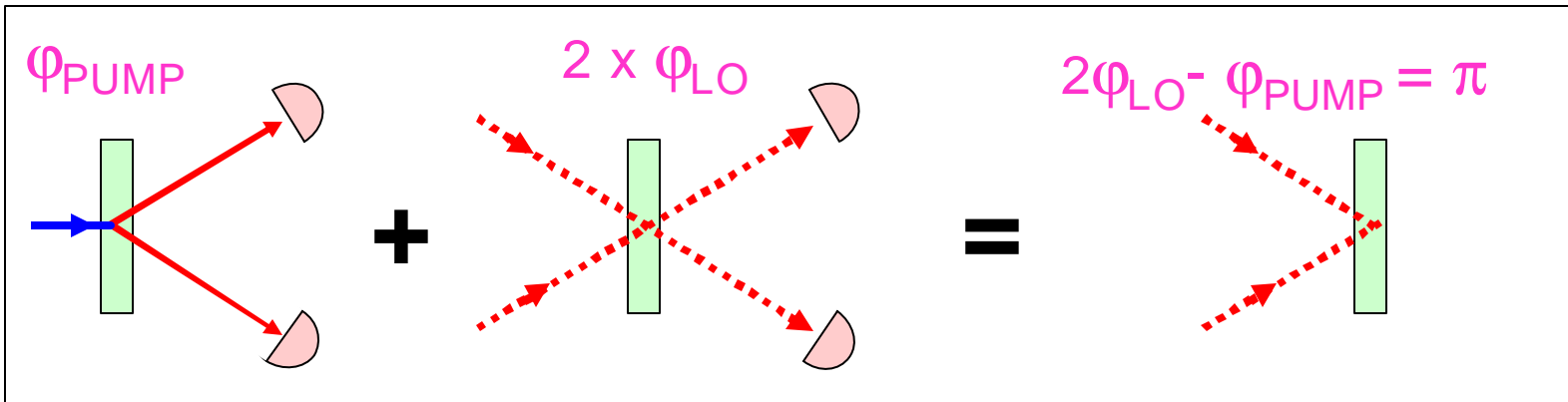
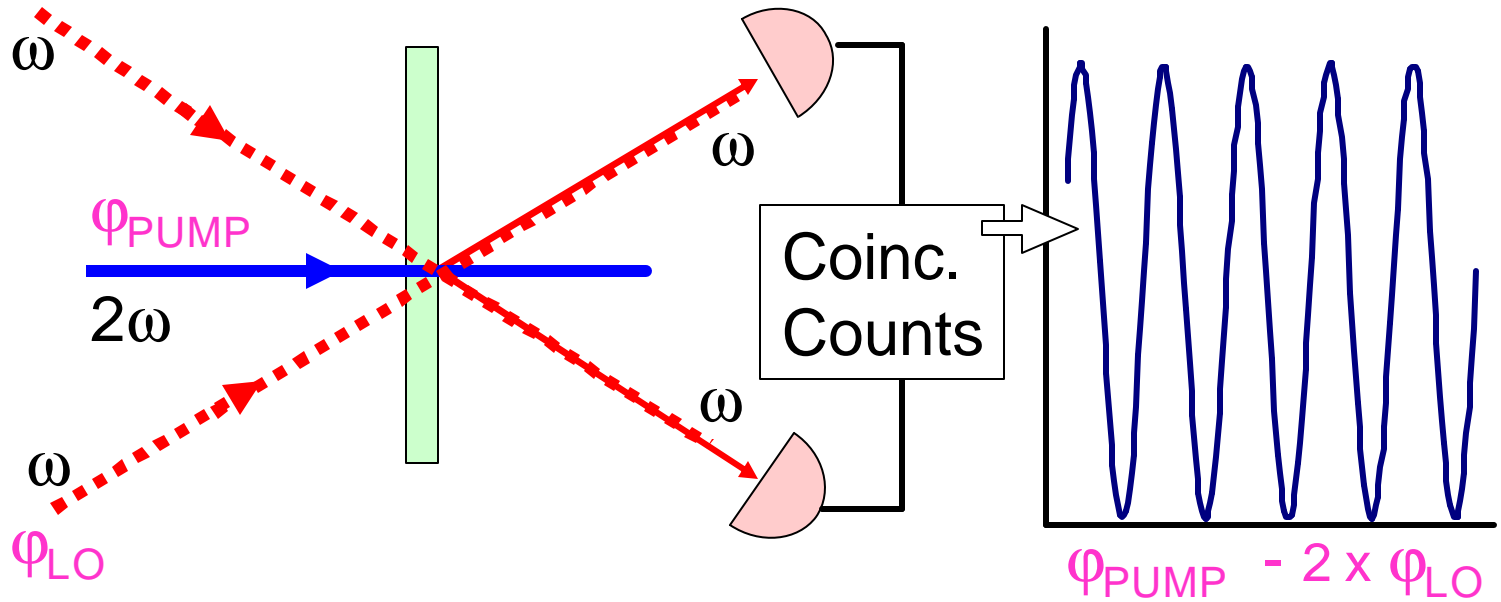
- A pump photon is spontaneously converted into two lower frequency photons in a material with a nonzero $\chi^{(2)}$



The Switch

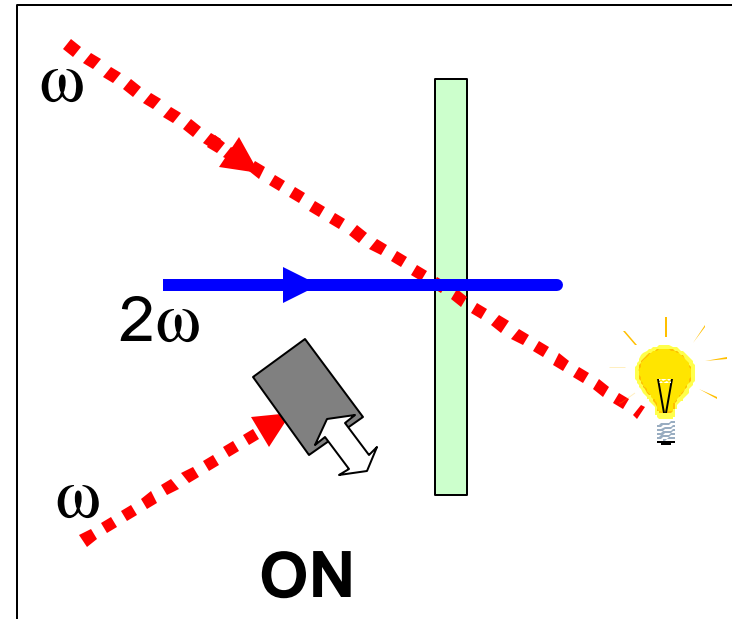
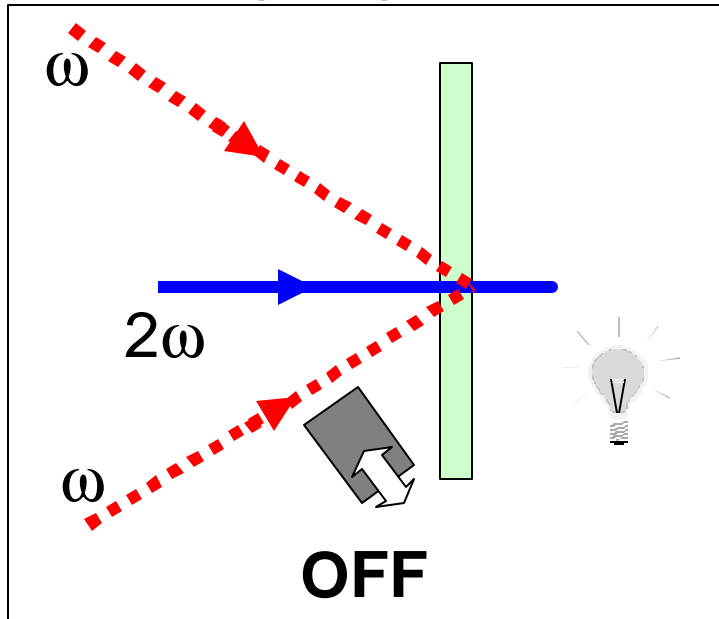


Φ_{LO} K. J. Resch, J. S. Lundeen, and A. M. Steinberg, Phys. Rev. Lett. **87**, 123603 (2001).



The Absorptive Gate

- Phase chosen so that all photon pairs are “absorbed” into the pump beam

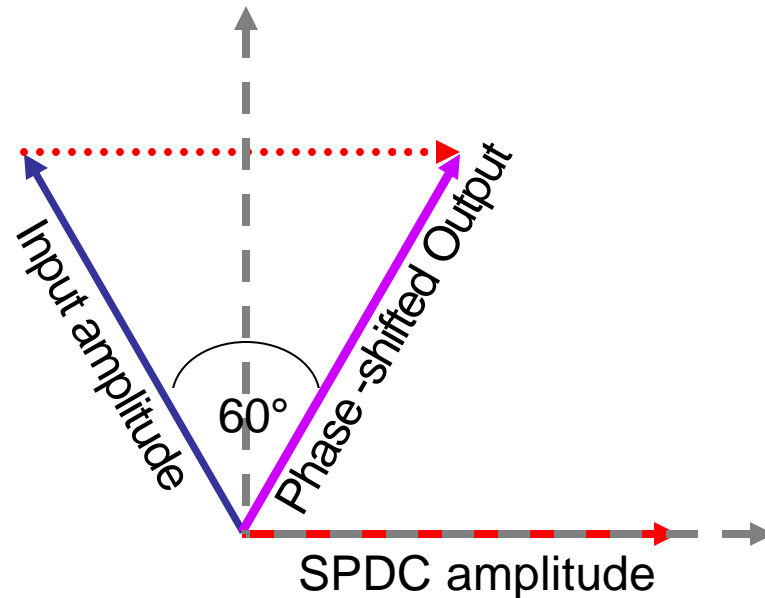


- On average < 1 photon per pulse
- One photon controls the transmission of the other beam
- The blue pump beam acts as a catalyst increasing SHG by a factor of 10^{10}



The Phase Gate

- Set two-photon amplitudes so that they add up to give a phase-shifted output



$$a|00\rangle + \beta|10\rangle + ?|01\rangle + d|11\rangle$$



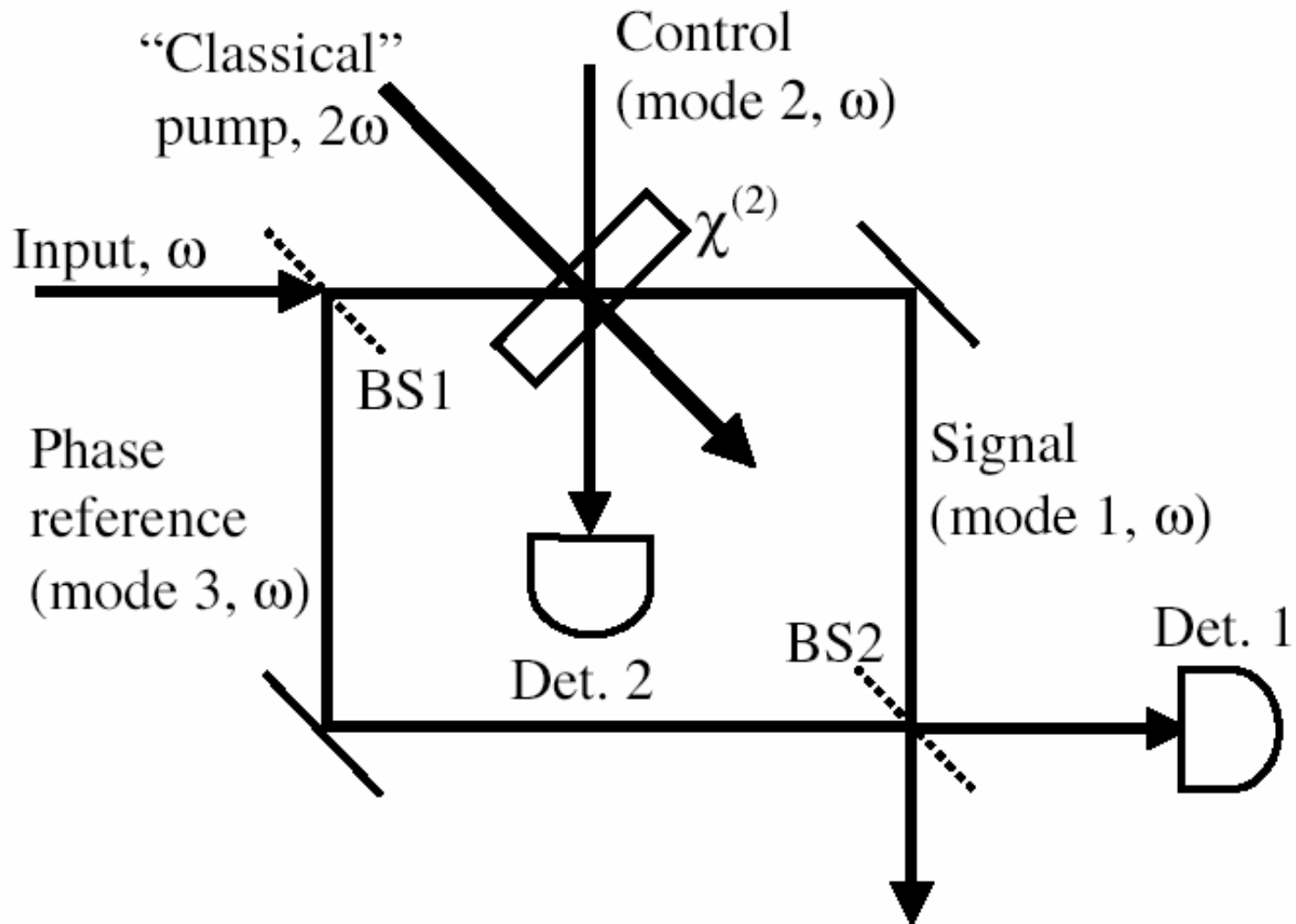
$$a|00\rangle + \beta|10\rangle + ?|01\rangle + de^{ip/3}|11\rangle$$

Resch et al, Phys. Rev. Lett. 89, 037914 (2002)

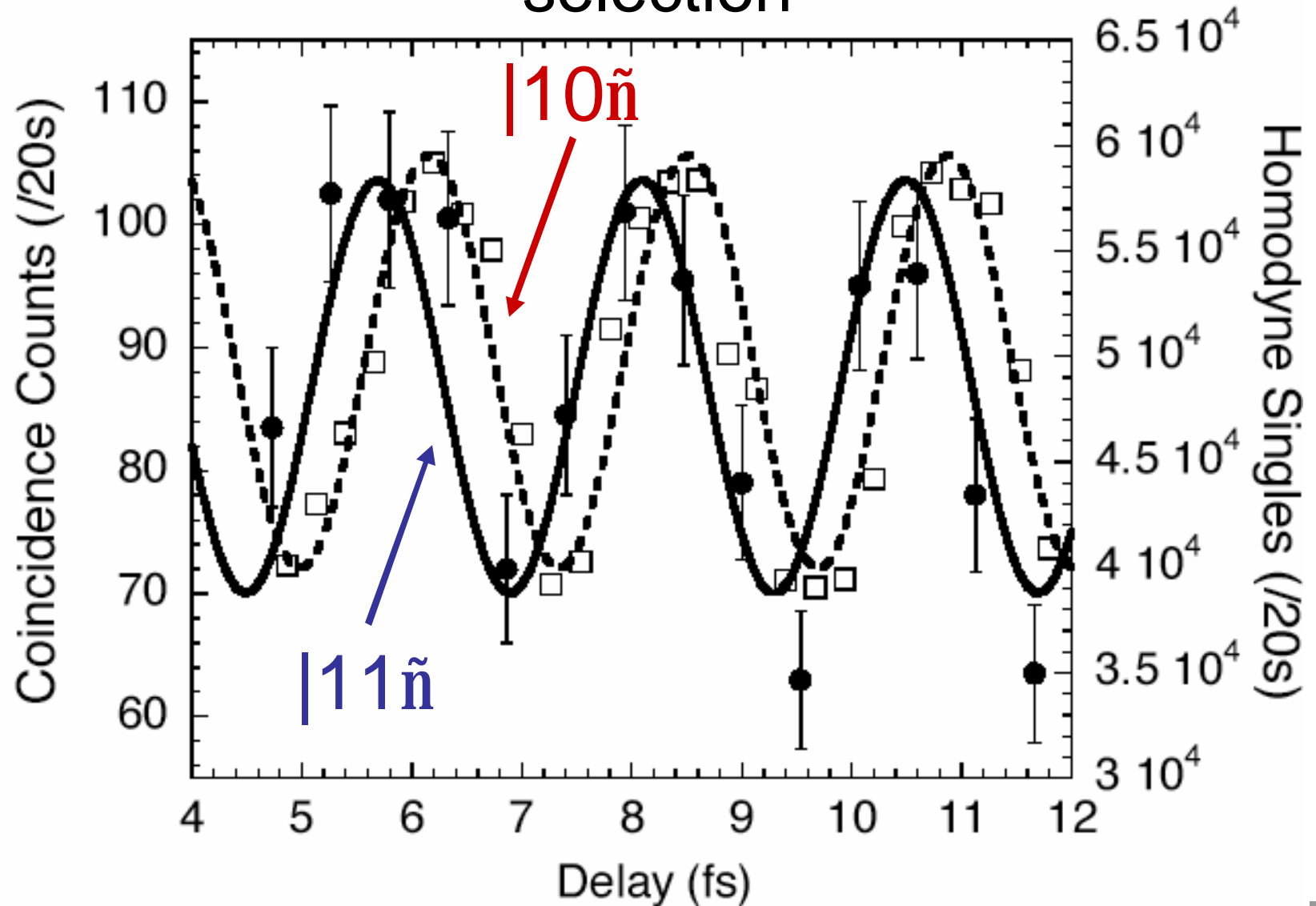


Measurement of Phase-shift

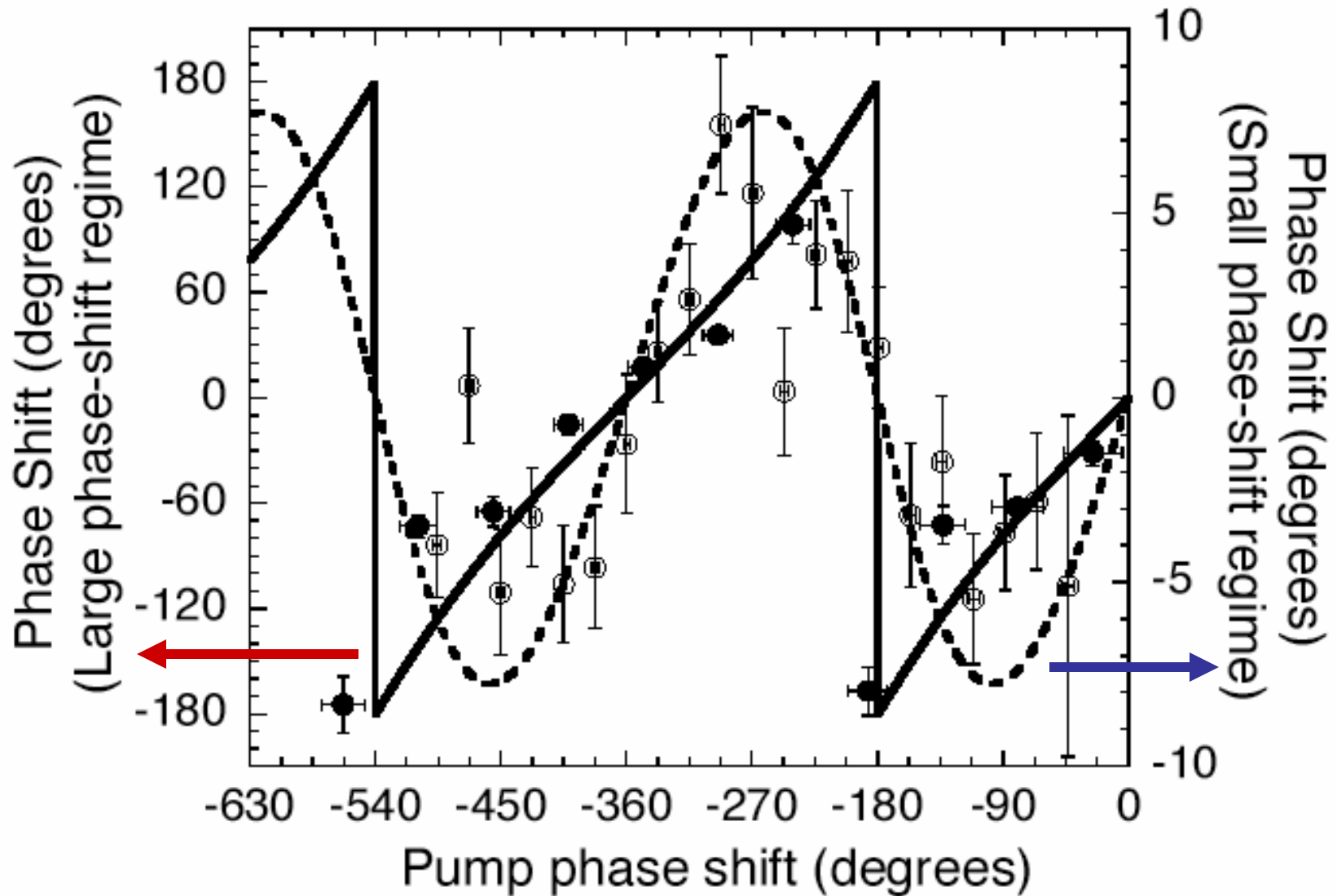
- Turn one of the input beams into a Mach-Zehnder and insert gate in one arm



Interference Fringes with and without Post-selection



Variable Phase-Shifts



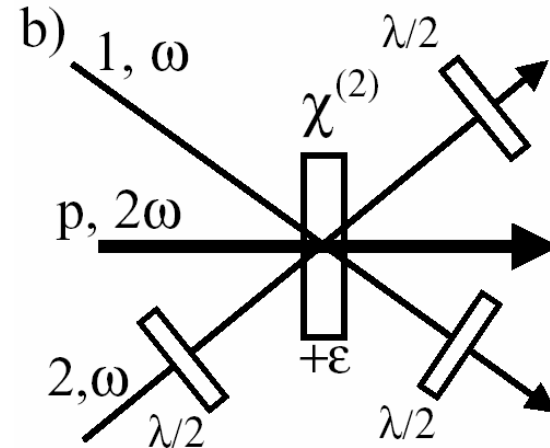
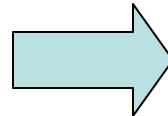
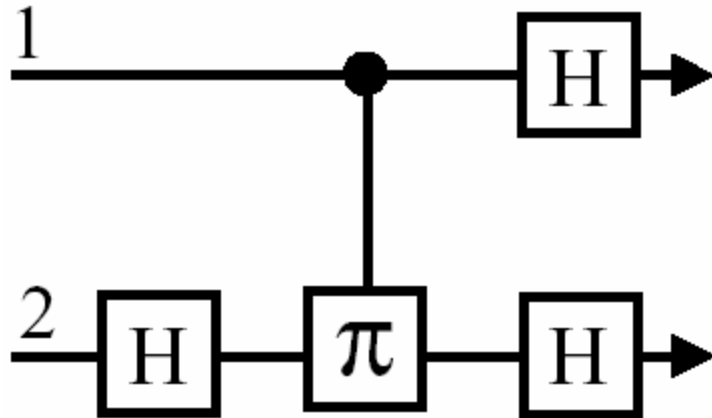
Caveats

- Typically, optical quantum computing uses single photons
- Single-photons do not have a well defined phase
- Both the absorptive gate and the phase gate rely on interference and hence require input beams with a well defined phase
- In practice: Input beams = weak coherent states or SPDC beams
- Concept: We can't know in advance whether the input beams contain a photon or not



Bell-state Analyzer

- Impossible to measure all four Bell-states with linear-optics



- Converts each Bell-state to a different basis state (i.e. $|\psi^\pm\rangle \rightarrow |HH\rangle$)
- Insert interference-based phase-gate in place of CPHASE
- Works for Dense-Coding (send 2 bits with one photon)
- Doesn't work for Teleportation

$$|0\rangle - \varepsilon |\psi^-\rangle \longrightarrow |0\rangle + \varepsilon |H\rangle_1 |H\rangle_2$$

$$|0\rangle - \varepsilon |\psi^+\rangle \longrightarrow |0\rangle + \varepsilon |H\rangle_1 |V\rangle_2$$

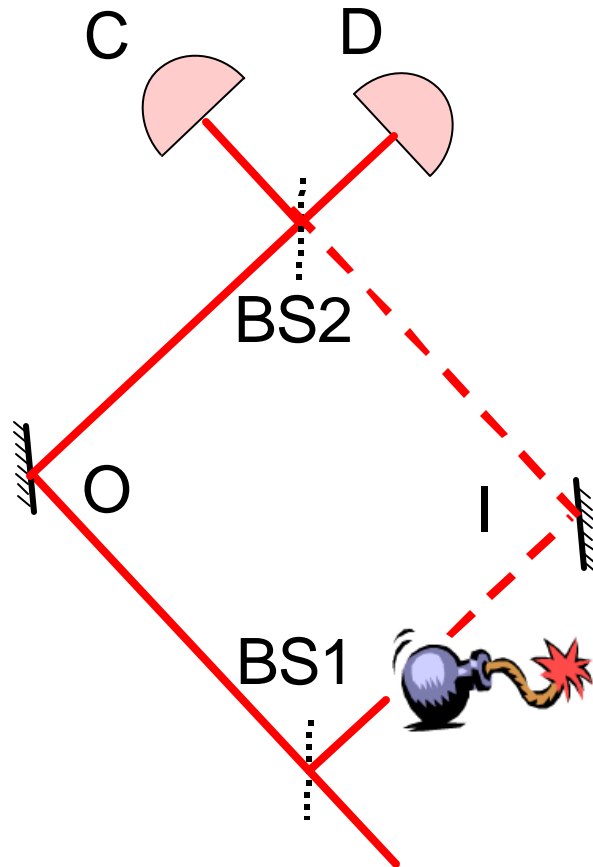
$$|0\rangle - \varepsilon |\phi^-\rangle \longrightarrow |0\rangle + \varepsilon |V\rangle_1 |H\rangle_2$$

$$|0\rangle - \varepsilon |\phi^+\rangle \longrightarrow |0\rangle + \varepsilon |V\rangle_1 |V\rangle_2$$



Interaction-Free Measurement

A. C. Elitzur, and L. Vaidman, Found. Phys. **23**, 987 (1993)



Bomb Absent:
Only detector C fires

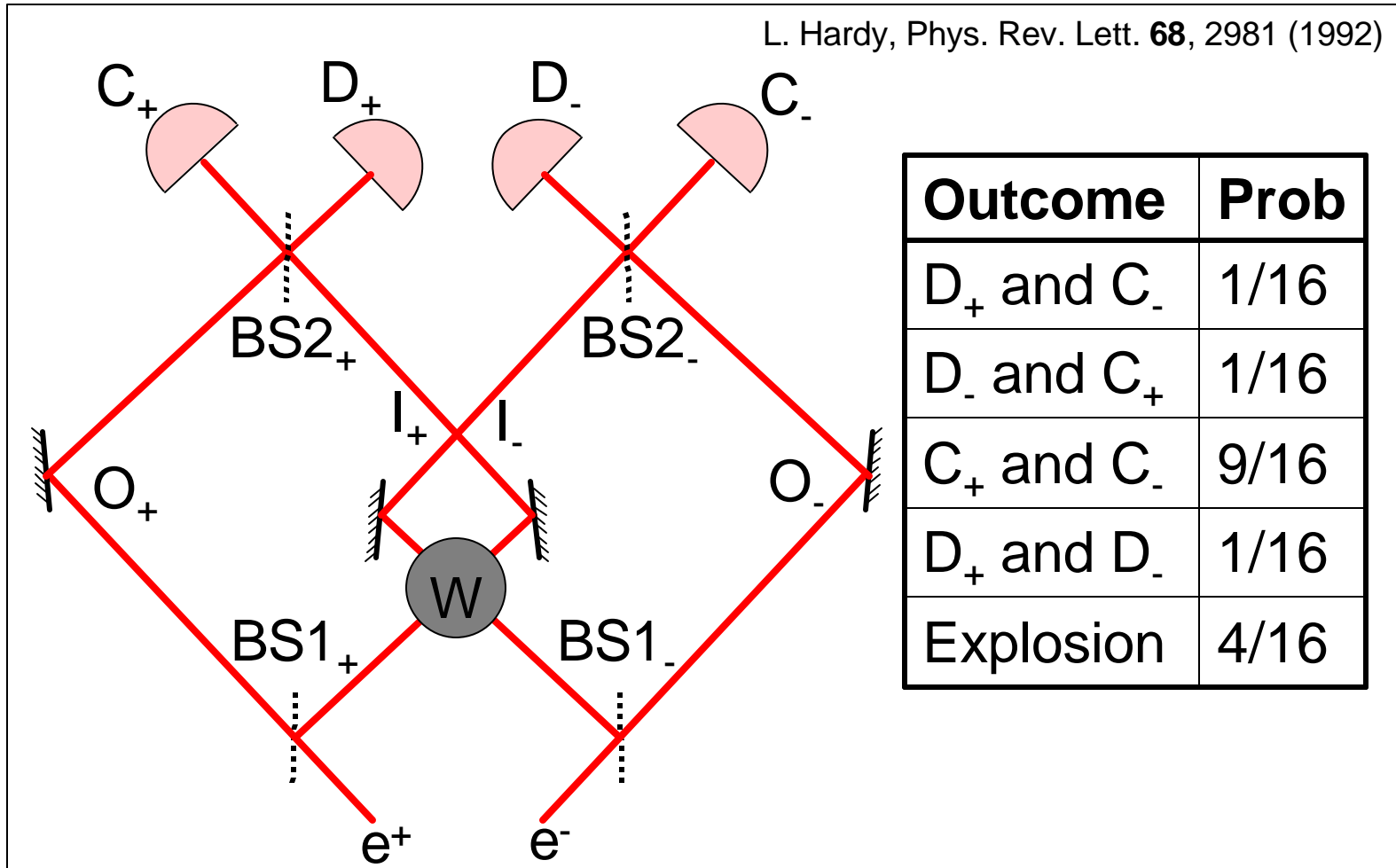
Bomb Present:

| Detector | Prob. | Result |
|----------|---------------|---------|
| C | $\frac{1}{4}$ | None |
| D | $\frac{1}{4}$ | Present |
| Neither | $\frac{1}{2}$ | Bang |



Hardy's Paradox

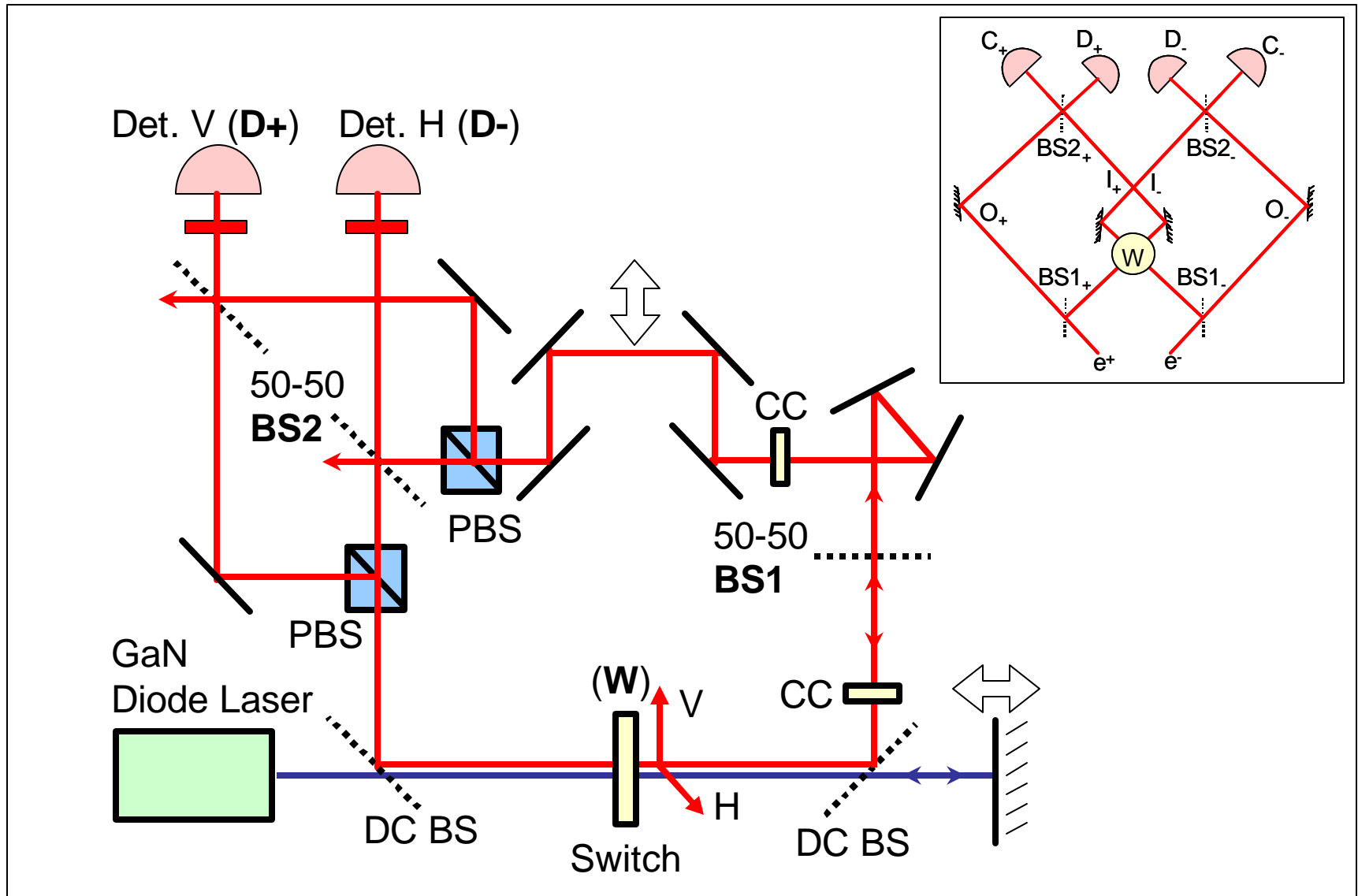
L. Hardy, Phys. Rev. Lett. **68**, 2981 (1992)



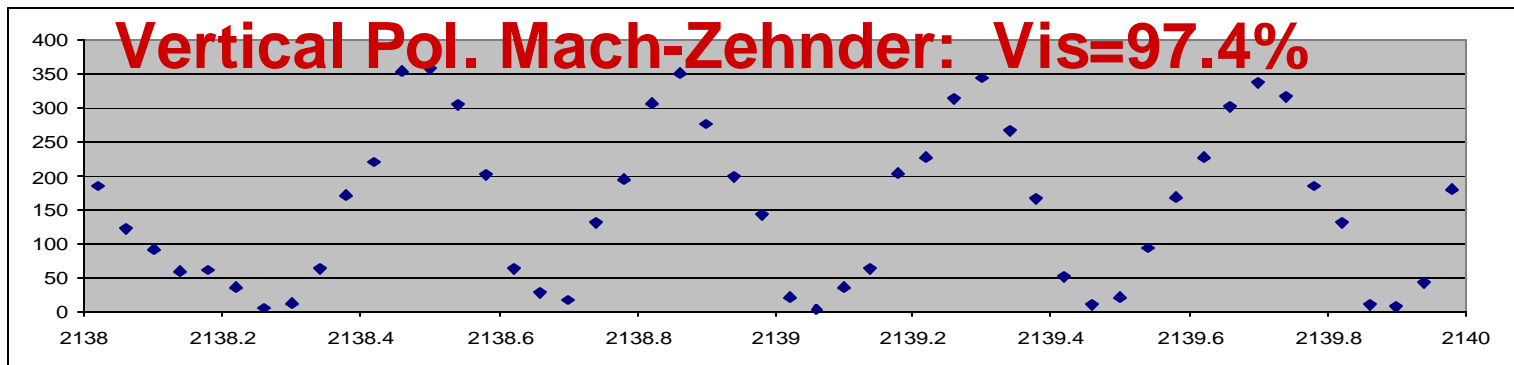
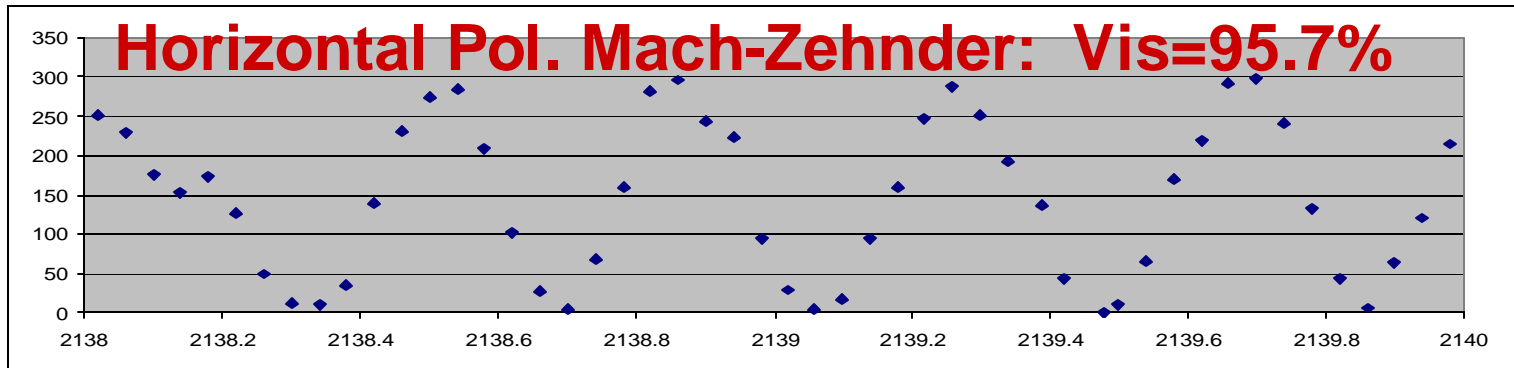
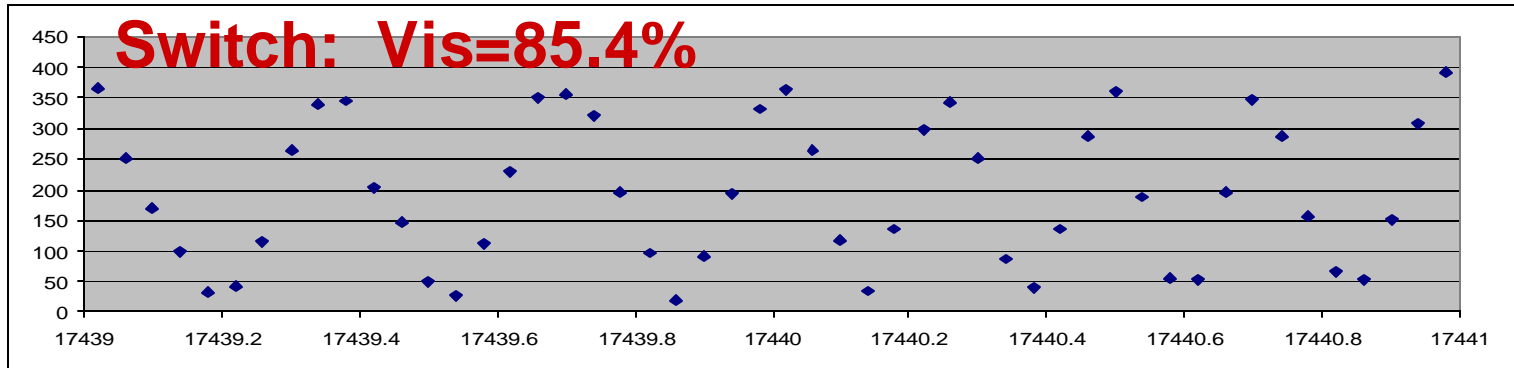
- Can we talk about the past in postselected QM?
- How should we interpret indirect quantum measurements?



Experimental Setup



Experimental Data



Experimental Data

Testing IFM+ If D+ clicks \mathcal{P}

| | |
|---------------------|-----|
| Photon is in arm I- | 96% |
| Photon is in arm O- | 4% |

Testing IFM- If D- clicks \mathcal{P}

| | |
|---------------------|-----|
| Photon is in arm I+ | 97% |
| Photon is in arm O+ | 3% |

Testing Switch

Rate of photon pairs in I+ and I-
= $10.4 \pm 0.33/5s$

The Paradox

Rate of D+ and D- coincidences
= $7.28 \pm 0.41/5s$

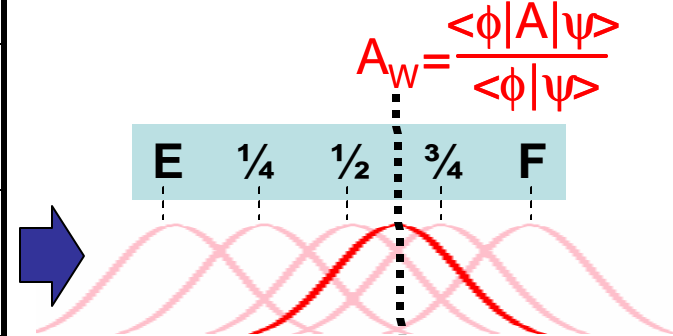


Weak Measurements

Aharonov, Albert, & Vaidman, PRL 60, 1351 ('88)

| Measurement | Pointer Position Uncertainty |
|-------------|--------------------------------|
| Ideal | Dirac Delta |
| Real | Width \ll Change in Position |
| Weak | Width \gg Change in Position |

Average position of pointer:
 $\text{Pointer}(X) = \exp[-(X - gA_W)^2 / \Delta X]$



$$\Delta X \Delta p \approx \hbar/2$$

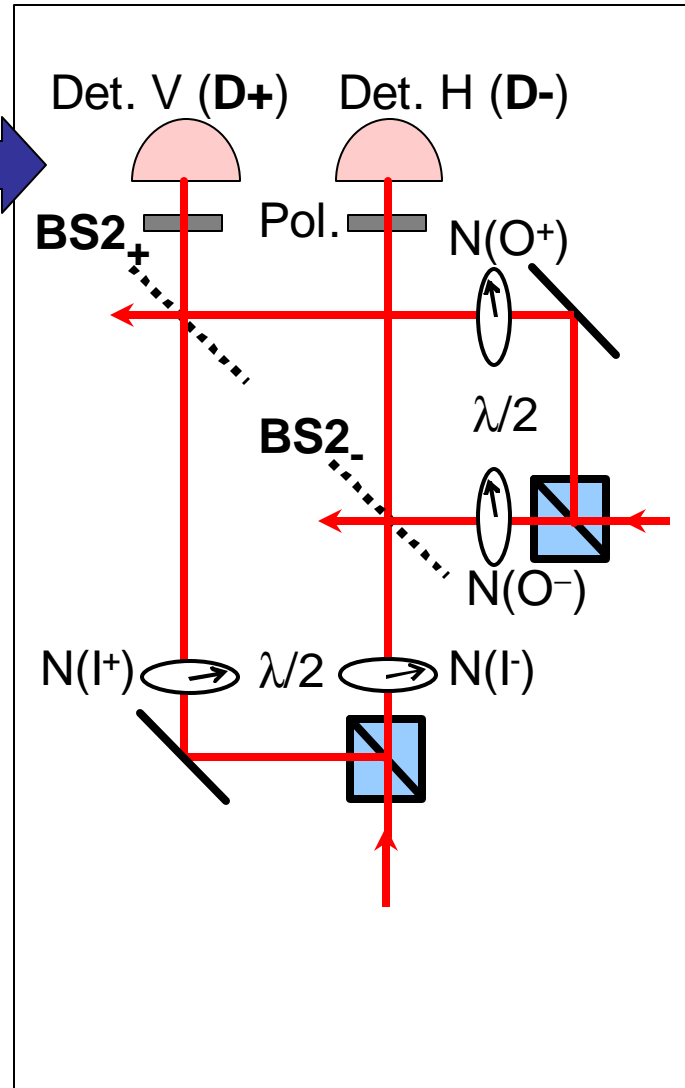
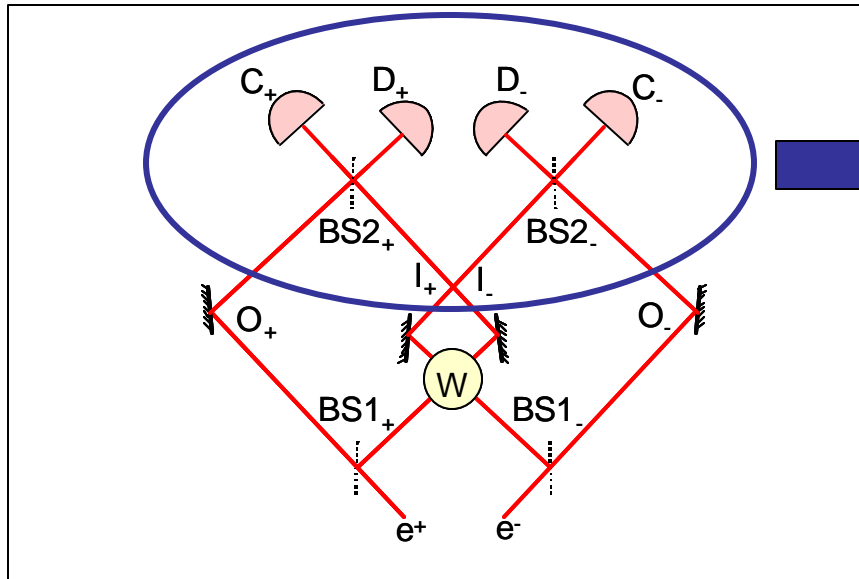
- \Rightarrow small disturbance
- \Rightarrow little system – pointer entanglement
- \Rightarrow simultaneous measurement of different weak values
- \Rightarrow useful for investigating post-selected systems: Hardy's Paradox



Weak Measurements in Hardy's Paradox

Y. Aharonov, A. Botero, S. Popescu, B. Reznik, J. Tollaksen, Phys. Lett. A 301, 130 (2001)

Resch & Steinberg, PRL 92, 130402 (2004)



| # In Arm | $N(I^-)$ | $N(O^-)$ | |
|----------|----------|----------|---|
| $N(I^+)$ | 0 | 1 | 1 |
| $N(O^+)$ | 1 | -1 | 0 |
| | 1 | 0 | |



Conclusions

- Interference-enhanced $\chi^{(2)}$ nonlinearities can be used to make absorptive and phase gates
- The phase-gate can be used to make a Bell-state analyzer useful for Dense-coding
- A single-photon level switch allows photons to annihilate each other with a high efficiency in Hardy's Paradox
- We are now experimenting with weak measurements in Hardy's Paradox.

