Comment on "Manipulating the frequency-entangled states by an acoustic-optical modulator"

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A recent theoretical paper by Shi *et al.* [Phys. Rev. A **61**, 064102 (2000)] proposes a scheme for entanglement swapping utilizing acousto-optic modulators without requiring a Bell-state measurement. In this Comment, we show that the proposal is flawed and no entanglement swapping can occur without measurement.

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I. INTRODUCTION

Entanglement swapping, a term coined by [1], is the process of creating entanglement between two particles that have never before interacted. When one generates entangled states, a pair of particles is typically created in tandem. The most common processes used to generate these entangled pairs are atomic cascades [2] and spontaneous parametric down-conversion [3,4]. In an entanglement swapping scheme, one begins with a *pair* of two-particle entangled pairs. In successful schemes to date [5], one performs a Bell measurement on two of the particles—one from each entangled pair. A successful Bell-state measurement collapses the remaining particles into a new entangled state—even though the particles have not directly interacted. The process of entanglement swapping was central to the experimental realization of quantum teleportation [6].

This Comment is on a recent proposal [7] for performing entanglement swapping with acousto-optic modulators (AOMs). There are two main results from this paper. The first is that entanglement swapping can be performed in the frequency domain. However, the AOM performs a transformation that is mathematically equivalent to using a simple beam splitter and this method is not different from other entanglement swapping techniques. In fact, the final states of interest [7] could be achieved trivially by using the technique of entanglement swapping used in [1] and frequency-shifting the desired spatial modes. The second result, which would represent new physics, is that entanglement swapping can occur without requiring a Bell measurement. The authors make a faulty assumption about the transformation an AOM performs on its input photon modes, which leads to incorrect conclusions. In this Comment, we describe generally how one should treat the interaction of an AOM with its two input light fields quantum mechanically. Then we apply this type of interaction to the proposed scheme, and show that no entanglement swapping can take place without the projective measurement stage. This is a general consequence of unitarity, and we discuss some of the relevant issues related to information transfer and causality.

II. THEORY

A. General theory

An acousto-optic modulator can be used to couple two modes of an electromagnetic field by means of a phonon field. A simple diagram from [7] shows this schematically (Fig. 1). The interaction between two input fields and an acousto-optic modulator will be described by an effective Hamiltonian, $\mathcal{H}_{\text{eff}}$, of the form

$$\mathcal{H}_{\text{eff}} = g b(\delta) a_t(\omega) a_d^\dagger(\omega+\delta) + g^* b^\dagger(\delta) a_t^\dagger(\omega) a_d(\omega+\delta), \tag{2.1}$$

where b and $b^\dagger$ are the annihilation and creation operators for the phonon field, a and $a^\dagger$ are the annihilation and creation operators for the photon field, and g is the coupling constant. The subscripts t and d refer to the mode labels shown in Fig. 1, and ω and $\omega+\delta$ are the photon frequencies. The first term in this Hamiltonian describes the destruction of a phonon of frequency δ, and a photon of frequency ω in mode t, and the creation of a photon with frequency $\omega+\delta$ in mode d. The second term in the Hamiltonian describes the creation of a phonon of frequency δ, and a photon of frequency ω in mode t, and the destruction of a photon in mode d. This Hamiltonian is manifestly Hermitian, and the propagator that follows from it must be unitary. If one assumes that the phonon field in the AOM is a classical field, which is a reasonable approximation for a coherent state of phonons with a high average phonon number, then we can replace the phonon operators with c numbers β and β^*. The Hamiltonian then becomes

$$\mathcal{H}_{\text{eff}} = g\beta a_t(\omega) a_d^\dagger(\omega+\delta) + g^*\beta^* a_t^\dagger(\omega) a_d(\omega+\delta). \tag{2.2}$$

Over an infinitesimal interaction time, dt, the AOM will perform the following transformations:

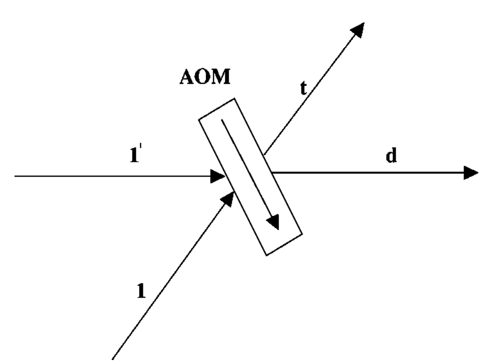

FIG. 1. The two input modes, 1 and 1′, enter an AOM and are converted to two output modes, t and d.

$$|\omega\rangle_1 \rightarrow |\omega\rangle_t - \frac{i}{\hbar} g\beta |\omega+\delta\rangle_d dt,$$

$$|\omega+\delta\rangle_{1'} \rightarrow |\omega+\delta\rangle_d - \frac{i}{\hbar} g^*\beta^* |\omega\rangle_t dt. \tag{2.3}$$

Over longer times, the transformation becomes

$$|\omega\rangle_1 \rightarrow \cos\left(\frac{|g\beta|t}{\hbar}\right)|\omega\rangle_t - i\frac{g\beta}{|g\beta|}\sin\left(\frac{|g\beta|t}{\hbar}\right)|\omega+\delta\rangle_d,$$

$$|\omega+\delta\rangle_{1'} \rightarrow \cos\left(\frac{|g\beta|t}{\hbar}\right)|\omega+\delta\rangle_d - i\frac{g^*\beta^*}{|g\beta|}\sin\left(\frac{|g\beta|t}{\hbar}\right)|\omega\rangle_t. \tag{2.4}$$

We can choose the interaction time to create equal superpositions of the outgoing modes and define the phase angle, $\phi = \arg(g\beta)$. The transformations then become

$$|\omega\rangle_1 \rightarrow \frac{1}{\sqrt{2}}[|\omega\rangle_t - ie^{i\phi}|\omega+\delta\rangle_d], \tag{2.5a}$$

$$|\omega+\delta\rangle_{1'} \rightarrow \frac{1}{\sqrt{2}}[|\omega+\delta\rangle_d - ie^{-i\phi}|\omega\rangle_t]. \tag{2.5b}$$

B. An AOM cannot perform entanglement swapping without a Bell measurement

The authors claim that an AOM (Fig. 1) can be modeled by taking two input modes, 1 and 1′, and transforming them to two output modes as follows:

$$|\omega\rangle_1 \xrightarrow{\text{AOM}} \frac{1}{\sqrt{2}}[|\omega\rangle_t + |\omega+\delta\rangle_d],$$

$$|\omega+\delta\rangle_{1'} \xrightarrow{\text{AOM}} \frac{1}{\sqrt{2}}[|\omega\rangle_t + |\omega+\delta\rangle_d]. \tag{2.6}$$

(The equations above actually differ from Eqs. (1) and (2) from [7] due to a presumed typographical error in the left side of the second equation, but are consistent with the rest of their paper.) However, such a transform is not allowed by quantum mechanics as it is nonunitary. The two input states are orthogonal, and must remain so by any unitary transformation. As one can see from the proposed transformation, the final states are not orthogonal—in fact they are identical. Such transformations destroy information and lead to paradoxes such as superluminal signaling. In the present case, if the proposed scheme were correct, a decision by Alice of whether or not to perform the AOM transformations could instantaneously affect a measurement by Bob of whether or not his photon pair was entangled.

Instead of the transformation given in [7], one should model the AOM by the unitary transformation described previously. We use the transforms from Eq. (2.5) and make the assumption that $\phi = 0$, without loss of generality. To put the transform into the same form as Eq. (2.6), Eq. (2.5b) is multiplied by a phase of $\exp(i\pi/2)$:

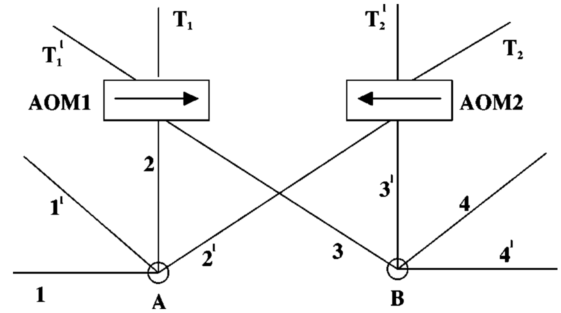

FIG. 2. The schematic for the proposed entanglement swapping scheme.

$$|\omega\rangle_1 \rightarrow \frac{1}{\sqrt{2}}[|\omega\rangle_t - i|\omega+\delta\rangle_d],$$

$$|\omega+\delta\rangle_{1'} \rightarrow \frac{1}{\sqrt{2}}[|\omega\rangle_t + i|\omega+\delta\rangle_d]. \tag{2.7}$$

The negative sign in the first term ensures that the final states remain orthogonal, preserving angle in the two-dimensional Hilbert space. We now follow through the calculations and describe the separate two-particle entangled states (see Fig. 2) as

$$|\phi\rangle = \frac{1}{\sqrt{2}}[|\omega\rangle_1|\omega+\delta\rangle_2 + |\omega+\delta\rangle_{1'}|\omega\rangle_{2'}],$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|\omega\rangle_3|\omega+\delta\rangle_4 + |\omega+\delta\rangle_{3'}|\omega\rangle_{4'}]. \tag{2.8}$$

The states $|\phi\rangle$ and $|\psi\rangle$ refer to the states of the particles created at the entangled-photon sources 1 and 2, respectively. The primed and unprimed subscripts refer to the spatial modes of the photons (Fig. 2), and the labels ω and $\omega+\delta$ refer to their angular frequencies. The two photons described in these states are not only entangled in their energy (frequency), but also in their spatial paths. We can now apply the following unitary transformations to the modes that interact with the AOMs in the scheme:

$$|\omega+\delta\rangle_2 \xrightarrow{\text{AOM1}} \frac{1}{\sqrt{2}}[|\omega\rangle_{T_{1'}} + i|\omega+\delta\rangle_{T_1}], \tag{2.9}$$

$$|\omega\rangle_3 \xrightarrow{\text{AOM1}} \frac{1}{\sqrt{2}}[|\omega\rangle_{T_{1'}} - i|\omega+\delta\rangle_{T_1}],$$

$$|\omega+\delta\rangle_{3'} \xrightarrow{\text{AOM2}} \frac{1}{\sqrt{2}}[|\omega\rangle_{T_2} + i|\omega+\delta\rangle_{T_{2'}}],$$

$$|\omega\rangle_{2'} \xrightarrow{\text{AOM2}} \frac{1}{\sqrt{2}}[|\omega\rangle_{T_2} - i|\omega+\delta\rangle_{T_{2'}}].$$

AOM1 and AOM2 simply refer to the transformation applied by the AOMs marked 1 and 2 in Fig. 2.

Using these transformations, the initial state describing the four photons, $|\phi\rangle\otimes|\psi\rangle$, will become

$$\begin{aligned}|\phi\rangle\otimes|\psi\rangle = \frac{1}{4}\{&[|\omega\rangle_1(|\omega\rangle_{T_{1'}} + i|\omega+\delta\rangle_{T_1}) + |\omega+\delta\rangle_{1'}(|\omega\rangle_{T_2}\\ &- i|\omega+\delta\rangle_{T_{2'}})]\otimes[|\omega+\delta\rangle_4(|\omega\rangle_{T_1'} - i|\omega+\delta\rangle_{T_1})\\ &+ |\omega\rangle_{4'}(|\omega\rangle_{T_2} + i|\omega+\delta\rangle_{T_{2'}})]\},\end{aligned} \tag{2.10}$$

$$\begin{aligned}= \frac{1}{4}[&|\omega\rangle_1|\omega+\delta\rangle_4(|\omega\rangle_{T_1'} + i|\omega+\delta\rangle_{T_1})\\ &\times(|\omega\rangle_{T_1'} - i|\omega+\delta\rangle_{T_1}) + |\omega+\delta\rangle_{1'}|\omega+\delta\rangle_4\\ &\times(|\omega\rangle_{T_2} - i|\omega+\delta\rangle_{T_{2'}})(\omega\rangle_{T_1'} - i|\omega+\delta\rangle_{T_1})\\ &+ |\omega\rangle_1|\omega\rangle_{4'}(|\omega\rangle_{T_1'} + i|\omega+\delta\rangle_{T_1})\\ &\times(|\omega\rangle_{T_2} + i|\omega+\delta\rangle_{T_{2'}}) + |\omega+\delta\rangle_{1'}|\omega\rangle_{4'}\\ &\times(|\omega\rangle_{T_2} - i|\omega+\delta\rangle_{T_{2'}})\\ &\times(|\omega\rangle_{T_2} + i|\omega+\delta\rangle_{T_{2'}})].\end{aligned} \tag{2.11}$$

The authors propose to discard the cases where both photons go through the same AOM (the first and fourth terms in the above equation), and are left with only the remaining two terms. These terms are

$$\begin{aligned}&|\omega+\delta\rangle_{1'}|\omega+\delta\rangle_4(|\omega\rangle_{T_2} - i|\omega+\delta\rangle_{T_{2'}})(|\omega\rangle_{T_1'} - i|\omega+\delta\rangle_{T_1})\\ &\quad + |\omega\rangle_1|\omega\rangle_{4'}(|\omega\rangle_{T_1'} + i|\omega+\delta\rangle_{T_1})(|\omega\rangle_{T_2} + i|\omega+\delta\rangle_{T_{2'}}).\end{aligned} \tag{2.12}$$

It is apparent that when the proper transformation is used, the terms describing the light after the AOM do *not* factor out and no entanglement swapping has occurred between photons 1 and 4. In fact, the other modes carry lem{complete} which-path information, totally decohering the two possible states of modes $1^{(\prime)}$ and $4^{(\prime)}$; one is left with purely classical correlations and no possibility of entanglement swapping.

In [7], a different AOM scheme is used to create a Greenberger-Horne-Zeilinger (GHZ) three-particle entangled state using a pair of two-photon entangled states. Unfortunately, the same transformation as shown in Eq. (1) is used to model the AOM. When the appropriate transformation is applied instead to their scheme, there is no three-particle entanglement.

III. CONCLUSION

The authors of [7] used a nonunitary transformation to describe the action of an AOM on a pair of input photon modes, and this appeared to lead to unconditional entanglement swapping. We have shown that when a unitary transformation is used instead, as required by quantum mechanics, no entanglement between the photons from different sources is achieved. Due to the same erroneous transformation, the claim that an AOM could create a GHZ state using a pair of two-photon entangled states is also incorrect. In general, no unitary transformation on one pair of photons can ever modify the reduced density matrix of a different pair. This is why effects such as entanglement swapping [5] and quantum teleportation [6] always require a nonunitary interaction (measurement).

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