Quantum State Preparation and Conditional Coherence

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It is well known that spontaneous parametric down-conversion can be used to probabilistically prepare single-photon states. We have performed an experiment in which *arbitrary superpositions* of zero- and one-photon states can be prepared by appropriate postselection. The optical phase, which is meaningful only for superpositions of photon number, is related to the relative phase between the zero- and one-photon states. Whereas the light from spontaneous parametric down-conversion has an undefined phase, we show that this technique collapses one beam to a state of well-defined optical phase when a measurement succeeds on the other beam.

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Quantum state preparation and characterization are of great importance for the new field of quantum information. The earliest examples of what today would be termed quantum state preparation date back to atomic cascade work in the 1970s for tests of Bell's inequalities [1,2]. Building on these techniques, single-photon states were created via postselection for interference experiments and Hanbury-Brown and Twiss-style experiments [3,4]. A great deal of effort has gone into designing sources of entangled photons with various desirable characteristics [5–7], and into generating and characterizing a variety of pure [8] or mixed states [9] of photon polarization. State preparation and tomography have also been studied in the context of NMR [10], micromaser [11], and ion trap experiments [12]. Quantum state preparation of light fields has acquired a new sense of urgency in light of a recent proposal for efficient quantum computation with linear optics [13]. While much current work is aimed at generating single photons on demand, at the moment the best source of single photons is spontaneous parametric down-conversion (SPDC), wherein detection of one photon of a down-converted pair guarantees the presence of the second [14]. A number of theoretical works, building on studies of nonlocality [15,16], have proposed and discussed methods for producing more complicated superpositions of number states called "truncated coherent states" [17–19]. We show experimentally how to produce arbitrary superpositions of photon number states $|0\rangle$ and $|1\rangle$ by a method considered by Hardy [15] and extended by Clausen *et al.* [17], and suggest a technique for varying the purity of these states as well.

The beams created by SPDC have essentially perfect number correlations which preclude the possibility of interference of a single down-converted beam with a phase reference. One could perform a phase measurement [20] of the signal mode in a homodyne apparatus by combining a phase-reference beam with the signal mode at a 50/50 beam splitter and detecting one of the outputs with a photon counting detector. To lowest order, there are two paths that lead to the detection of a photon. Either a down-converted photon is detected, or a homodyne photon is detected. However, the presence or absence of a photon in the idler mode provides "which-path" information that distinguishes the two Feynman paths and destroys the interference; this implies that a single down-converted beam has no phase. It has been shown experimentally that it is possible to induce a well-defined *relative* phase between two signal beams from two different down-conversion crystals by creating spatiotemporal overlap between their idler modes [21,22], such that they no longer contain any information regarding which crystal emitted the signal photon. In this experiment, we instead use the intrinsic number uncertainty of a coherent state to weaken the perfect number correlations between the down-converted modes. We show that overlapping a weak coherent state with the idler mode from SPDC and conditioning on the detection of a photon in the idler mode collapses the signal beam to a superposition of $|0\rangle$ and $|1\rangle$ with a well-defined *absolute* optical phase. We measure that phase via homodyne measurement with a weak local oscillator [23–25]. This preparation is thus similar to the third-order ("wave-particle") correlations recently studied in cavity QED [26].

Figure 1a shows a simplified schematic of our experiment. A strong laser with frequency 2ω is used to pump a $\chi^{(2)}$ nonlinear crystal and create pairs of photons via degenerate SPDC into the signal and idler modes. The weak coherent state, with frequency ω, passes through the nonlinear crystal into the idler mode. The idler terminates at a single-photon detector (detector 1), and the signal mode is superposed with a local oscillator for homodyne detection. One output mode of the homodyne terminates on another photon detector (detector 2). Although the signal photons arise entirely from SPDC, which is known to have no well-defined phase, detection of a photon at detector 1 collapses the signal into a state which can interfere (perfectly, in principle) with the weak local oscillator; in this sense, postselection has induced a well-defined optical phase in the signal. Figure 1b shows the two lowest-order paths that lead to the detection of a photon in the idler mode. The photon can come from a down-conversion pair, in which case there is also a photon in the signal mode; alternatively, the photon can come from the weak coherent state, in which case there are no photons in the signal mode. When detector 1 fires, a coherent superposition of these two possibilities is created, leaving the signal mode in a well-defined superposition of $|0\rangle$ and $|1\rangle$.

We consider the experiment with a simple three-mode theory. The signal and idler modes will be treated quantum mechanically, and the strong pump laser classically. The signal mode is initially in the vacuum state, while the idler is in a weak ($|\alpha| \ll 1$) coherent state. The initial state of the system can be written, $|\psi\rangle = |0\rangle_s \otimes |\alpha\rangle_i \approx |0\rangle_s \otimes (|0\rangle_i + \alpha|1\rangle_i)$. Normalization has been suppressed for clarity, and the coherent state has been written to first order in α. This state can be propagated forward in time under the $\chi^{(2)}$ interaction in the nonlinear crystal by the interaction Hamiltonian, $\mathcal{H}_{\text{int}} = g a_s^\dagger a_i^\dagger a_p + g^* a_s a_i a_p^\dagger \approx g\gamma a_s^\dagger a_i^\dagger + g^*\gamma^* a_s a_i$, in which g is a coupling constant, γ is the amplitude of the classical pump field, and $a_{s,i}^{(\dagger)}$ are the lowering (raising) operators for the signal and idler modes. By first-order perturbation theory, the state will evolve to

$$|\psi(t)\rangle \approx \left(1 - \frac{it}{\hbar}\mathcal{H}_{\text{int}}\right)|\psi\rangle = |\psi\rangle - \frac{it}{\hbar}(g\gamma a_s^\dagger a_i^\dagger + g^*\gamma^* a_s a_i)\,|0\rangle_s(|0\rangle_i + \alpha|1\rangle_i)$$

$$\approx |0\rangle_s|0\rangle_i + \left(\alpha|0\rangle_s - \frac{it}{\hbar}g\gamma|1\rangle_s\right)|1\rangle_i\,, \qquad (1)$$

where we have neglected higher-order terms ($g\gamma\alpha$, etc.). It can be seen from this expression that the signal and idler modes are nonmaximally entangled. If there are no photons in the idler, then there are also no photons in the signal. If, however, there is a photon in the idler mode, then the signal mode is collapsed to a specific superposition of $|0\rangle$ and $|1\rangle$. The optical phase is the phase difference between the quantum amplitudes for neighboring photon number states — not to be confused with an overall phase factor that has no physical significance. The amplitude for zero signal photons is determined by the amplitude of the weak coherent state; the amplitude for one signal photon is determined by the amplitude for SPDC.

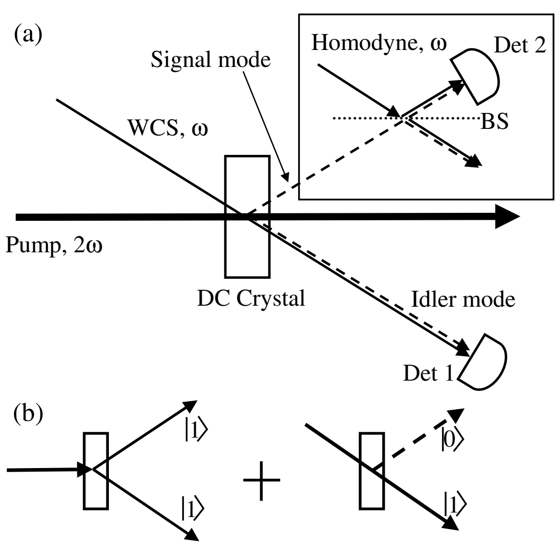

FIG. 1. (a) A simplified cartoon of the experiment. A strong pump laser of frequency 2ω creates photon pairs into the signal and idler modes via SPDC. A weak coherent state (WCS) with frequency ω passes through the nonlinear crystal such that it is superposed with the idler mode. The signal mode is studied via homodyne measurement with a second weak coherent beam. Single-photon counters are placed in the idler mode and in one output of the homodyne setup. (b) The two lowest-order paths that lead to a photon in the idler mode. Either a down-conversion event can occur, in which case there is a single signal photon, or an idler photon can come from the coherent state, in which case there are no photons in the signal mode.

Figure 2 is a more detailed schematic of our experiment. The phase relationships between the pump laser, the weak coherent state, and the homodyne beam are ensured as they all originate from the same oscillator — a femtosecond Ti:Sa laser operating at an 80 MHz rep rate with a center wavelength of 810 nm. The homodyne and weak coherent state beams are created by separating a small amount of the fundamental beam at beam splitters BS 1 and BS 2 and highly attenuating it with neutral density filters until there is on average 1 photon per 100 pulses. The pump for down-conversion is created by frequency doubling the remaining fundamental light from the Ti:Sa. The fundamental beam is removed from the second harmonic by use of colored glass filters. We performed the experiment using type-II degenerate collinear down-conversion (in contrast with Fig. 1 where type-I noncollinear SPDC was implied for clarity) in a 0.5-mm β-barium borate (BBO) nonlinear optical crystal. The weak coherent state is vertically polarized and is mixed with the pump laser at BS3 so that it passes through the crystal and overlaps the idler mode. The polarizing beam splitter (PBS) separates the vertically

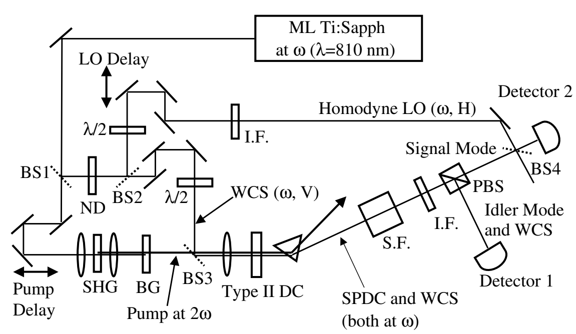

FIG. 2. Detailed schematic of the setup. BS1-4 are 90/10 (T/R) beam splitters; SHG consists of 2 lenses and a type-I phase-matched 0.1-mm BBO crystal; BG is a colored glass filter; S.F. is a spatial filter; I.F. is a 10-nm bandwidth interference filter; PBS is a polarizing beam splitter; $\lambda/2$ is a half-wave plate; ND are neutral density filters; type II DC is a 0.5-mm BBO crystal phase-matched for type-II down-conversion; detectors 1 and 2 are single-photon counting modules.

polarized idler mode and weak coherent state from the horizontally polarized signal mode. The idler and weak coherent state impinge on detector 1 and the signal mode is mixed with the phase reference at BS4 for a homodyne measurement with detector 2 at one of the output ports. Both detectors are single-photon counting modules (Perkin Elmer SPCM-AQR-13). The polarization of the weak homodyne beam is rotated to horizontal by a half-wave plate prior to BS4.

In order for interference to occur, we must make the weak coherent state and down-conversion beam completely indistinguishable. We therefore postselect a single spatial mode with a spatial filter, and use a 10-nm interference filter centered at 810 nm to increase the spectral overlap of the beams. The spatial filter consists of a 25-μm pinhole followed by a 2-mm iris and a 5-cm lens, placed one focal length beyond the pinhole. In order to increase the flux of down-converted light created in a single mode, we focus the pump to a spot on the down-conversion crystal and then image that spot onto the spatial filter pinhole [27]. We observe interference by varying two different spatial delay lines. As shown in Fig. 2, optical delays in the homodyne path (LO delay) and the pump laser arm (pump delay) can be independently controlled by use of motorized translation stages.

To maximize the interference visibility at the homodyne detector, the intensity of the homodyne beam must match the intensity of the signal beam when an idler photon is detected (i.e., the *conditional* intensity). The conditional probability to find a photon in the signal beam is given by the expectation value for the photon number in the state shown in Eq. (1) when an idler photon is detected,

$$\langle n \rangle = \frac{(\frac{t}{\hbar})^2 |g|^2 |\gamma|^2}{|\alpha|^2 + (\frac{t}{\hbar})^2 |g|^2 |\gamma|^2} . \qquad (2)$$

We treat the homodyne beam as a weak coherent state of the form $|0\rangle + \beta|1\rangle$, and use the approximation $|\alpha|^2 \gg (\frac{t}{\hbar})^2 |g|^2 |\gamma|^2$, since in our experiment the counting rate from the weak coherent state is much higher than that from SPDC. We require

$$|\beta|^2 = \frac{(\frac{t}{\hbar})^2 |g|^2 |\gamma|^2}{|\alpha|^2} . \qquad (3)$$

This condition is fulfilled when the coincidence counting rate from the pair of classical beams is equal to the coincidence counting rate from SPDC. In our experiment, the coincidence rate from down-conversion was $10.2 \pm 0.3\ \mathrm{s}^{-1}$, and the coincidence rate from the two classical beams (the weak coherent state and homodyne beam) was $19.1 \pm 0.5\ \mathrm{s}^{-1}$—this discrepancy lowers the maximum visibility to 94%. The singles rates from down-conversion were 718 ± 4 and $958 \pm 4\ \mathrm{s}^{-1}$ at detectors 1 and 2, respectively. The singles rate at detector 2 from the homodyne beam was approximately $3.4 \times 10^4\ \mathrm{s}^{-1}$ and the singles rate from the weak coherent state at detector 1 was $5.2 \times 10^4\ \mathrm{s}^{-1}$. (Because of the uncorrelated nature of the classical light sources, the classical beams must be much brighter than the down-conversion beams to obtain the same coincidence rates.) Based on the coincidence rate from down-conversion and the singles rate at detector 1 from the weak coherent state, the conditional intensity at detector 2 was approximately $1.6 \times 10^4\ \mathrm{s}^{-1}$. It should also be noted that due to the polarization geometry we used, the LO beam also yielded about $150\ \mathrm{s}^{-1}$ singles counts at detector 1 due to imperfect extinction. Figure 3 shows the coincidence rate as a function of the delay in the pump laser arm (Fig. 3a) and the homodyne arm (Fig. 3b). The visibilities of the fringes in both cases are about 32% due to imperfect spectral, temporal, and spatial mode matching. The fringe spacing in Fig. 3b—where the delay is introduced in the pump arm—is half of that in Fig. 3a, where the delay is introduced in the homodyne arm. This is as one would expect, since the wavelength of the light in the pump arm is half of that in the homodyne arm. The fringes were subject to a large amount of phase drift due to the large spatial size of the interferometer; this limited counting times.

Fringes can also be observed in the singles rates at detector 2 in both Fig. 3a and Fig. 3b, but for different reasons.

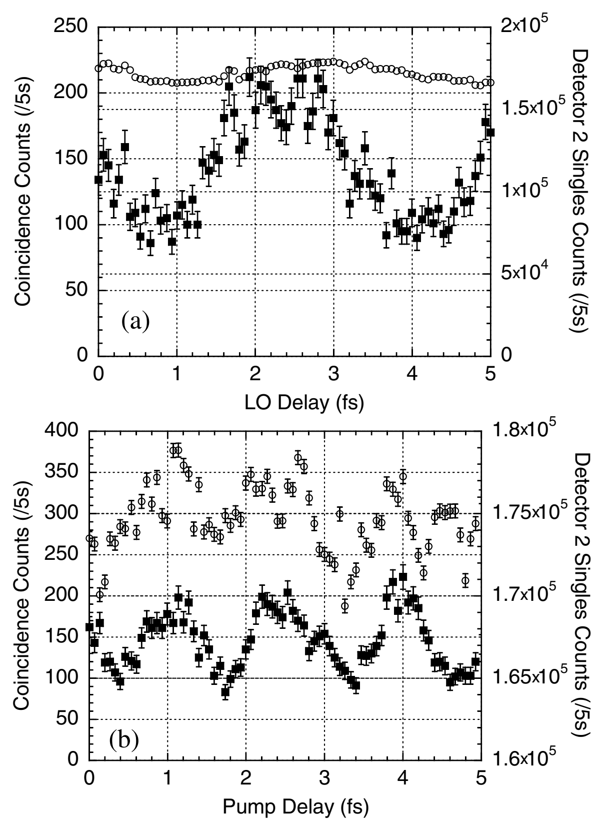

FIG. 3. The coincidence rate (solid squares) and singles rate (open circles) as a function of (a) the homodyne delay or (b) the pump delay. The coincidence visibilities are both 32%. The singles fringes in (a) are caused by spurious classical interference effect due to imperfect extinction at the PBS, but the singles fringes in (b) are not subject to the same spurious effect and are due to a partial phase imparted by the weak coherent state.

The singles rate fringe visibility in Fig. 3a, when the homodyne delay was changed, is about 4%. These fringes are caused by imperfect extinction of the weak coherent state at the PBS. The light that manages to leak through the beam splitter forms an effective Mach-Zehnder interferometer with the homodyne laser leading to a spurious classical interference effect. Notice that the fringe patterns are not in phase. No such spurious effect can occur when the pump arm is translated because no 810-nm light leaks through that arm. Figure 3b shows the singles rate as a function of pump arm delay. There is evidence for singles fringes at the 1.0%–1.5% level. In contrast with the spurious classical interference effect, these fringes are phase-locked to the coincidence fringes, and correspond to the period of 405 nm light. Examining Eq. (1), it is apparent that the state of the light in the signal beam, when tracing out the state of the idler, is in a mixed state of vacuum and a coherent state with a well-defined phase. This small admixture of a coherent state leads to a very small fringe visibility in the singles rate *unconditionally*. Conditioning on the detection of an idler photon projects out the coherent state, leading to high visibility fringes observable in the coincidence rate. It is well known that quantum and classical theories of light have common predictions for intensity. These very small singles (intensity) fringes are exactly what one would expect from a classical treatment of the experiment. We have already discussed the fact that one beam from SPDC cannot interfere with a phase reference: however, light created via *stimulated* down-conversion [or difference frequency generation (DFG)] can [28]. The intensity fringes can be interpreted as interference between a very small amplitude for DFG and the phase reference [27]. The low visibility is due to a large spontaneous background. However, in the classical theory, the coincidence rate would merely be the product of the singles rates. The product of a flat singles rate and a 1.5% fringe pattern has a fringe visibility of only 1.5%, which is 20 times smaller than the coincidence visibility observed experimentally. A quantum mechanical treatment is required to explain the strong correlations.

We have demonstrated that the signal beam created via spontaneous parametric down-conversion can be made coherent (in the sense that it can interfere with a phase reference) conditioned on the detection of a single photon. This was accomplished by overlapping a weak coherent state with the idler beam through the down-conversion crystal and conditioning on photon detection in the idler mode. Accompanying the coherence in the conditional intensity at detector 2 (coincidence rate) is an unconditional intensity effect at detector 2. While the latter effect can be understood from purely classical nonlinear optical theory, the much larger coincidence visibility cannot. The conditional coherence technique is an example of creating asymmetric entanglement (entangling different degrees of freedom on different beams of light) and can be used to create arbitrary superpositions of zero and one photon. A theoretical proposal has shown how this technique can be cascaded to produce more complicated superpositions with higher numbers of photons [17]. By varying the temporal overlap between the pump laser and the weak coherent state, it is possible to adapt this scheme to produce mixed or partially mixed states as well. Further development of these techniques is important for manipulating the electromagnetic field at the most fundamental level.

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