Total Reflection Cannot Occur with a Negative Delay Time

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***Abstract*—It was recently predicted [1] that the frustrated Gires–Tournois interferometer exhibits a negative delay time for reflection. Given its 100% reflectivity, this appears to contradict causality. We demonstrate that an additional, positive, contribution comes from consideration of the transverse dimension. We prove that this contribution is always large enough to enforce a positive total delay.**

***Index Terms*—Dielectric materials, Fabry–Perot interferometers, Gires–Tournois interferometer, Goos–Hänchen shift, group delay times, nonhomogenous media, tunneling.**

I. Introduction

NEGATIVE group delays or superluminal velocities have been predicted and shown experimentally in systems where particles have a very small transmission probability, and in the presence of active media. These experiments are not in conflict with relativity or our ideas of causality, for different reasons [2]. If the particle has a very small probability of transmission, as in the case of tunneling or frustrated total internal reflection [3]–[6], then the superluminal propagation can be understood as a preferential transmission of the leading edge of the wavepacket. Thus, no energy need ever be transmitted faster than the speed of light. If the system is an active medium, as in the case of light propagating in an inverted gas of cesium [7], [8], then the system can selectively amplify the leading edge of the pulse and reduce the trailing edge of a wavepacket provided that the pulse has a smoothly varying envelope. This results in a time advancement of the smooth pulse, again without any energy travelling faster than c. A sharp disturbance, which is representative of a true signal, would not exhibit the same superluminal velocity. In all cases, information travels slower than c, a smooth pulse being reconstructed by Taylor expansion of the incident pulse [9], [2]. Apparent superluminal propagation has been reported for a Bessel beam in free space [10], but proper consideration of the 3-D energy flow in this situation makes it clear that the experiment can be understood by considering the (subluminal) diffraction of different components of the beam. A recent calculation [1] has predicted that the group delay for a Gaussian beam in a frustrated Gires–Tournois interferometer can be negative. What makes the Gires–Tournois interferometer, and total internal reflection, in general, a unique problem, is that the superluminal behavior is predicted to occur for 100% of the incident light, and since the system is made entirely of dielectrics, it is completely passive. In this work, we show that a negative delay will not occur once the positive time contribution from the Goos–Hänchen shift is included. While it can be shown that the delay time, including the contribution from the Goos–Hänchen shift, is superluminal in the more optically dense media, it is still unknown whether the propagation can be superluminal in the lowest-index material. In all specific cases we have analyzed, the total propagation is, in fact, slower than the speed of light in the lowest index material, suggesting no problem with relativistic causality.

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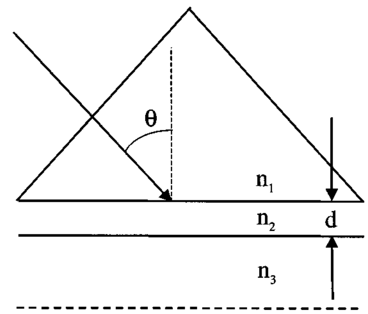

Fig. 1. Schematic drawing of the frustrated Gires–Tournois interferometer. A negative group delay time is predicted for the case where $n_1 > n_3 > n_2$, and the light is incident in the first region at an angle greater than $\theta = \sin^{-1}(n_3/n_1)$. The first and third layers are bulk dielectric materials, and the middle layer has a thickness d. The first layer is drawn as a prism to allow the light to be coupled into it. The dashed line means the third layer is semi-infinite.

A Gires–Tournois interferometer is essentially a Fabry–Perot cavity with a 100% back reflector. The Frustrated Gires–Tournois interferometer considered in this paper is made entirely of dielectrics and the 100% reflectivity is ensured by the process of total internal reflection (TIR), meaning that the light will always be incident beyond the critical angle for total internal reflection from the final layer. The "Gires–Tournois time" (τ_{GT}) for reflection was calculated in [1] by following stationary phase theory and differentiating the reflection phase shift with respect to the incident frequency. The Gires–Tournois time describes the time delay between the incident field at the origin reaching its maximum and the outgoing field at the same point reaching its maximum. Due to the Goos–Hänchen shift [11], however, the reflected peak never crosses the origin. Thus, the delay for the 2-D peak of the pulse to leave the surface may differ from τ_{GT}. We show in this paper that the Goos–Hänchen shift contributes an extra positive time which is always large enough to make the total time delay positive.

The frustrated Gires–Tournois interferometer is a simple three-layer stack of dielectric materials (Fig. 1). The first and

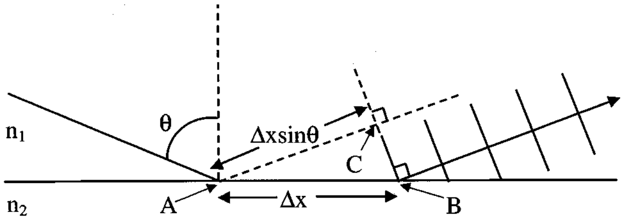

Fig. 2. Sketch of light undergoing TIR. For TIR, $n_1 > n_2$ and $\theta > \sin^{-1}(n_2/n_1)$. The peak of the outgoing pulse leaves a distance, Δx, along the interface from where the peak of the incident pulse hit. The field reaches a maximum at the point A at the time calculated by Tournois, $t_{\rm GT}$. The peak of the outgoing pulse leaves point B, due to the Goos–Hänchen shift, at a time $t_{\rm GT} + (n_1/c)\Delta x \sin\theta$ (equal to the time at which the wavefront from A reaches point C, at velocity c/n_1).

third layers are bulk dielectrics with indices n_1 and n_3. The middle layer is a dielectric with index n_2 and has a thickness d. The light impinges from the first layer with an angle of incidence greater than the critical angle θ_c, for the first and third layers (i.e., $\sin\theta_c = n_3/n_1$). While other combinations of indices were considered in [1], negative Gires–Tournois times were predicted to occur only when $n_1 > n_3 > n_2$. Since $n_3 > n_2$, the light is ensured to be incident beyond the critical angle for the first and second layers as well. Practically, one can couple the light into, and out of, the interferometer by using a prism for the first dielectric layer.

II. Physical Origin of the Gires–Tournois and the Goos–Hänchen Times

Fig. 2 shows a beam undergoing reflection from an interface at an angle of incidence beyond critical. In the diagram, Δx is the Goos–Hänchen shift and $k_x\Delta x$ is the phase accumulated during that shift, where k_x is the x-component of the wavevector. The total phase shift, $\phi_{\rm TOT}$, upon reflection in TIR can be written as

$$\phi_{\rm TOT} = \phi_R + k_x\Delta x \tag{1}$$

$$= \phi_R + \frac{n_1\omega}{c}\sin\theta\Delta x \tag{2}$$

where

- ω angular frequency of the light;
- n_1 index of refraction of the first medium;
- θ angle of incidence;
- c speed of light in vacuum;
- ϕ_R reflection phase shift for an incident plane wave—the phase shift in Tournois's calculation of the delay.

Using stationary phase theory, we can calculate the partial derivative of the total phase with respect to the angular frequency and set it to zero to calculate the phase time or the group delay, τ

$$\frac{\partial(\phi_{\rm TOT} - \omega\tau)}{\partial\omega} = 0 = \frac{\partial\phi_R}{\partial\omega} + \frac{n_1}{c}\sin\theta\Delta x - \tau \tag{3}$$

$$\tau = \frac{\partial\phi_R}{\partial\omega} + \frac{n_1}{c}\sin\theta\Delta x. \tag{4}$$

We can see that the total group delay consists of two components. The first term is the time calculated by Tournois in [1], and is due to the nontrivial phase shift from the multilayer structure. The second term is an additional contribution not considered in the original proposal, and is referred to in this work as the Goos–Hänchen time $\tau_{\rm GH}$. The spatial shift arises from the phase shifts experienced by different plane-wave components of a bounded beam. We can apply stationary phase theory again to find the size of this shift from the phase by taking the partial derivative of the total phase with respect to θ and setting it to zero

$$\frac{\partial\phi_{\rm TOT}}{\partial\theta} = 0 = \frac{\partial\phi_R}{\partial\theta} + \frac{n_1\omega}{c}\cos\theta\Delta x \tag{5}$$

$$\Delta x = -\frac{c}{n_1\omega\cos\theta}\frac{\partial\phi_R}{\partial\theta}. \tag{6}$$

This allows us to write out the Goos–Hänchen time in terms of the phase factor that has already been calculated as

$$\tau_{\rm GH} = \frac{n_1}{c}\sin\theta\left(-\frac{c}{n_1\omega\cos\theta}\frac{\partial\phi_R}{\partial\theta}\right) \tag{7}$$

$$= -\frac{\tan\theta}{\omega}\frac{\partial\phi_R}{\partial\theta}. \tag{8}$$

As one can see from Fig. 2, the Goos–Hänchen time is exactly the amount of time it would take a light wavefront to move a distance $\Delta x \sin\theta$. The transverse displacement of the beam does not affect the delay time for any individual wavefront, but does modify the time delay for appearance of the spatiotemporal peak of the pulse. Since interferometric techniques are not sensitive to transverse shifts of the wavefront, such measurements will yield only the Gires–Tournois time, not the full two-dimensional delay.

III. Positivity of the Total 2-D Time

In the sections to come, we prove that the total reflection time is greater than zero. We will start by writing the time delay in terms of the Gires–Tournois time from [1] and the Goos–Hänchen time for which we derive an expression. We show that an inequality can be written in a useful form with a single term on the left-hand side which must be less than the sum of three terms on the right-hand side. We then analyze these three terms in turn.

The positivity of the total 2-D time, including the delay contribution due to the Goos–Hänchen shift ($\tau_{\rm GH}$) in the frustrated Gires–Tournois interferometer (Fig. 1), can be expressed as the inequality

$$\tau_{\rm TOT} = \tau_{\rm GH} + \tau_{\rm GT} > 0 \tag{9}$$

where $\tau_{\rm TOT}$ is the total delay time.

A. Introduction of the Specific Form of the Gires–Tournois Time and the Goos–Hänchen Times

We use the following definitions for the complex cosines, α_2 and α_3:

$$\alpha_2 = \left(\frac{n_1^2}{n_2^2}\sin^2\theta - 1\right)^{1/2} \tag{10}$$

$$\alpha_3 = \left(\frac{n_1^2}{n_3^2}\sin^2\theta - 1\right)^{1/2} \tag{11}$$

which are proportional to the decay constants of the evanescent waves set up in the second and the third materials. The angle of incidence is θ, and the indices for the ith region are written n_i. To be consistent with the previous calculation [1], we define the following terms:

$$\tan\left(\frac{\psi_{12,p}}{2}\right) = \frac{n_1\alpha_2}{n_2\cos\theta} \tag{12}$$

$$\tan\left(\frac{\psi_{12,s}}{2}\right) = \frac{n_2\alpha_2}{n_1\cos\theta} \tag{13}$$

$$\tau' = \frac{n_2 d}{c}\,\alpha_2 \tag{14}$$

$$\tanh(\gamma_p) = \frac{n_2\alpha_3}{n_3\alpha_2} \tag{15}$$

$$\tanh(\gamma_s) = \frac{n_3\alpha_3}{n_2\alpha_2}. \tag{16}$$

The terms $\psi_{12,\{s,p\}}$ are the TIR phase shifts at the n_1–n_2 interface for s and p-polarization, respectively. τ' and $\gamma_{s,p}$ are useful substitutions. From [1], we use the expression for the phase of a reflected wave for angles of incidence, θ, beyond the critical angle for the first and third layers

$$\tan\frac{\phi_{s,p}}{2} = -\tan\left(\frac{\psi_{12,s,p}}{2}\right)\tanh(\omega\tau' + \gamma_{s,p}). \tag{17}$$

This expression for the phase is only valid in the range of angles from $\theta_c < \theta < \pi/2$, and for the interferometer where $n_1 > n_3 > n_2$. The group delay for a wavepacket can be calculated using stationary phase theory. That delay, which we called the Gires–Tournois time, was also derived in [1] and is given by the following expression:

$$\tau_{\mathrm{GT}} = \left.\frac{\partial\phi_{s,p}}{\partial\omega}\right|_\theta = \frac{-2\tan\left(\frac{\psi_{12,s,p}}{2}\right)\left[1-\tanh^2(\omega\tau'+\gamma_{s,p})\right]\tau'}{1+\tan^2\left(\frac{\psi_{12,s,p}}{2}\right)\tanh^2(\omega\tau'+\gamma_{s,p})}. \tag{18}$$

The additional contribution to the reflection time due to the Goos–Hänchen shift occurs in any two dimensional system undergoing total internal reflection. This Goos–Hänchen time delay was described in [12] and is given by the following expression as in (19) and (20), shown at the bottom of the page.

B. Converting the Inequality in (9) to a Useful Form

The inequality in (9) can be rewritten as

$$-\tau_{\mathrm{GT}} < \tau_{\mathrm{GH}}. \tag{21}$$

Upon substitution of (18) and (20), the inequality becomes (22), as shown at the bottom of the page, and can be converted to a more desirable form

$$\frac{\omega\tau'}{\tan\theta} < \frac{\frac{1}{2}\left(1+\tan^2\left(\frac{\psi_{12,s,p}}{2}\right)\right)\frac{\partial\psi_{12,s,p}}{\partial\theta}\tanh(\omega\tau'+\gamma_{s,p})}{\tan\left(\frac{\psi_{12,s,p}}{2}\right)\left[1-\tanh^2(\omega\tau'+\gamma_{s,p})\right]} + \omega\frac{\partial\tau'}{\partial\theta} + \frac{\partial\gamma_{s,p}}{\partial\theta}. \tag{23}$$

$$\tau_{\mathrm{GH}} = \frac{-\tan\theta}{\omega}\left.\frac{\partial\phi_{s,p}}{\partial\theta}\right|_\omega \tag{19}$$

$$= \frac{2\tan\theta\left[\begin{array}{l}\frac{1}{2}\left(1+\tan^2\left(\frac{\psi_{12,s,p}}{2}\right)\right)\frac{\partial\psi_{12,s,p}}{\partial\theta}\tanh(\omega\tau'+\gamma_{s,p}) \\ +\tan\left(\frac{\psi_{12,s,p}}{2}\right)\left[1-\tanh^2(\omega\tau'+\gamma_{s,p})\right]\left(\omega\frac{\partial\tau'}{\partial\theta}+\frac{\partial\gamma_{s,p}}{\partial\theta}\right)\end{array}\right]}{\omega\left[1+\tan^2\left(\frac{\psi_{12,s,p}}{2}\right)\tanh^2(\omega\tau'+\gamma_{s,p})\right]} \tag{20}$$

$$\frac{-2\tan\left(\frac{\psi_{12,s,p}}{2}\right)\left[1-\tanh^2(\omega\tau'+\gamma_{s,p})\right]\tau'}{1+\tan^2\left(\frac{\psi_{12,s,p}}{2}\right)\tanh^2(\omega\tau'+\gamma_{s,p})} < \frac{2\tan\theta\left[\begin{array}{l}\frac{1}{2}\left(1+\tan^2\left(\frac{\psi_{12,s,p}}{2}\right)\right)\frac{\partial\psi_{12,s,p}}{\partial\theta}\tanh(\omega\tau'+\gamma_{s,p}) \\ +\tan\left(\frac{\psi_{12,s,p}}{2}\right)\left[1-\tanh^2(\omega\tau'+\gamma_{s,p})\right]\left(\omega\frac{\partial\tau'}{\partial\theta}+\frac{\partial\gamma_{s,p}}{\partial\theta}\right)\end{array}\right]}{\omega\left[1+\tan^2\left(\frac{\psi_{12,s,p}}{2}\right)\tanh^2(\omega\tau'+\gamma_{s,p})\right]} \tag{22}$$

For this step to be true, $\tan\theta$ and $\tan(\psi_{s,p}/2)$ must be positive so as not to change the direction of the inequality. In the range of angles $\theta_c < \theta < \pi/2$, $\tan\theta$ is positive, and $\tan(\psi_{s,p}/2)$ is defined in terms of all positive quantities and is, thus, also positive. To simplify (23) further, we use the fact that $1+\tan^2(\psi_{12,s,p}/2) > 1$ to write the new inequality

$$\frac{\omega\tau'}{\tan\theta} < \frac{\frac{1}{2}\frac{\partial\psi_{12,s,p}}{\partial\theta}\tanh(\omega\tau'+\gamma_{s,p})}{\tan\left(\frac{\psi_{12,s,p}}{2}\right)\left[1-\tanh^2(\omega\tau'+\gamma_{s,p})\right]} + \omega\frac{\partial\tau'}{\partial\theta} + \frac{\partial\gamma_{s,p}}{\partial\theta}. \tag{24}$$

Equation (24) is a sufficient condition to establish the positivity of τ_{TOT}. We can make the following definitions:

$$A = \frac{\omega\tau'}{\tan\theta} \tag{25}$$

$$B = \frac{\frac{1}{2}\frac{\partial\psi_{12,s,p}}{\partial\theta}\tanh(\omega\tau'+\gamma_{s,p})}{\tan\left(\frac{\psi_{12,s,p}}{2}\right)\left[1-\tanh^2(\omega\tau'+\gamma_{s,p})\right]} \tag{26}$$

$$C = \omega\frac{\partial\tau'}{\partial\theta} \tag{27}$$

$$D = \frac{\partial\gamma_{s,p}}{\partial\theta} \tag{28}$$

which allows us to write (24) as

$$A < B + C + D. \tag{29}$$

C. Proof that A is less than C

Term A is less than term C in (29) if the following inequality is satisfied:

$$A < C \tag{30}$$

$$\frac{\omega\tau'}{\tan\theta} < \omega\frac{\partial\tau'}{\partial\theta}. \tag{31}$$

Substituting the definition for τ' [(14)] into (31) and taking the derivative yields

$$\left(\frac{n_2\omega d}{c}\right)\frac{\left(\frac{n_1^2}{n_2^2}\sin^2\theta-1\right)^{1/2}}{\tan\theta} < \left(\frac{n_2\omega d}{c}\right) \times\left[\frac{n_1^2\sin\theta\cos\theta}{n_2^2\left(\frac{n_1^2}{n_2^2}\sin^2\theta-1\right)^{1/2}}\right] \tag{32}$$

$$\left(\frac{n_1^2}{n_2^2}\sin^2\theta-1\right) < \frac{n_1^2}{n_2^2}\sin^2\theta. \tag{33}$$

This inequality is thus satisfied and term A is less than term C.

D. Proof of Positivity of Terms B and D

Term B can be written as follows:

$$\frac{\frac{1}{2}\frac{\partial\psi_{12,s,p}}{\partial\theta}\tanh(\omega\tau'+\gamma_{s,p})}{\tan\left(\frac{\psi_{12,s,p}}{2}\right)\left[1-\tanh^2(\omega\tau'+\gamma_{s,p})\right]}. \tag{34}$$

The definitions for ψ_{12} and γ differ for s and p-polarization; thus, we will consider each polarization separately. For p-polarization, B becomes

$$\frac{\frac{1}{2}\frac{\partial\psi_{12,p}}{\partial\theta}\tanh(\omega\tau'+\gamma_{p})}{\tan\left(\frac{\psi_{12,p}}{2}\right)\left[1-\tanh^2(\omega\tau'+\gamma_{p})\right]} \tag{35}$$

$$= \left[\frac{\frac{1}{\alpha_2}\frac{\partial\alpha_2}{\partial\theta}\frac{1}{\cos\theta}+\tan\theta}{1+\left(\frac{n_1\alpha_2}{n_2\cos\theta}\right)^2}\right]\times\frac{\tanh(\omega\tau'+\gamma_p)}{\left[1-\tanh^2(\omega\tau'+\gamma_p)\right]} \tag{36}$$

$$= \left[\frac{\frac{n_1^2}{n_2^2}\frac{\sin\theta}{\alpha_2^3}+\tan\theta}{1+\left(\frac{n_1\alpha_2}{n_2\cos\theta}\right)^2}\right]\times\frac{\tanh(\omega\tau'+\gamma_p)}{\left[1-\tanh^2(\omega\tau'+\gamma_p)\right]}. \tag{37}$$

The numerator of the first fraction is positive in the region of interest $\theta_c < \theta < \pi/2$, as is the denominator. The denominator of the second fraction is positive as $0 < \tanh^2 x < 1$. We can show the numerator of the second fraction is positive by proving the inequality

$$\tanh(\omega\tau'+\gamma_p) > 0 \tag{38}$$

which is equivalent to showing that

$$\omega\tau'+\gamma_p > 0 \tag{39}$$

which we rewrite as

$$\frac{n_2\omega d}{c}\alpha_2 + \arctan\left(\frac{n_2\alpha_3}{n_3\alpha_2}\right) > 0. \tag{40}$$

Both of these terms are positive in the region $\theta_c < \theta < \pi/2$, as the argument of the inverse tangent is positive. Thus, term B is positive for p-polarization. For s-polarization, B is

$$\frac{\frac{1}{2}\frac{\partial\psi_{12,s}}{\partial\theta}\tanh(\omega\tau'+\gamma_{s})}{\tan\left(\frac{\psi_{12,s}}{2}\right)\left[1-\tanh^2(\omega\tau'+\gamma_{s})\right]} = \left[\frac{\frac{1}{\alpha_2}\frac{\partial\alpha_2}{\partial\theta}\frac{1}{\cos\theta}+\tan\theta}{1+\left(\frac{n_2\alpha_2}{n_1\cos\theta}\right)^2}\right]\times\left\{\frac{\tanh(\omega\tau'+\gamma_s)}{\left[1-\tanh^2(\omega\tau'+\gamma_s)\right]}\right\}. \tag{41}$$

The numerator for the first fraction is identical to that for p-polarization and is thus positive. The denominator for the first fraction is also positive as all the components are real in the region of interest. The denominator of the second fraction is positive, again because $0 < \tanh^2 x < 1$. The final numerator can be shown to be positive again by considering the argument of the inverse hyperbolic tangent in the inequality

$$\omega\tau' + \gamma_s > 0 \tag{42}$$

$$\frac{n_2\omega d}{c}\,\alpha_2 + \arctan\left(\frac{n_3\alpha_3}{n_2\alpha_2}\right) > 0. \tag{43}$$

Again, both terms are positive in the region $\theta_c < \theta < \pi/2$. Therefore, B is also positive for s-polarization over the relevant range of angles. Term D from (29) can be proven to be positive by showing

$$\frac{\partial\gamma_{s,p}}{\partial\theta} > 0. \tag{44}$$

We begin by proving the relation for p-polarization

$$\frac{\partial\gamma_p}{\partial\theta} > 0 \tag{45}$$

which can be expanded as

$$\frac{\dfrac{n_2}{n_3\alpha_2}\dfrac{\partial\alpha_3}{\partial\theta} - \dfrac{n_2\alpha_3}{n_3\alpha_2^2}\dfrac{\partial\alpha_2}{\partial\theta}}{1 - \left(\dfrac{n_2\alpha_3}{n_3\alpha_2}\right)^2} > 0. \tag{46}$$

This inequality can be shown to hold by demonstrating that both the numerator and the denominator are positive. We begin with the denominator and show

$$1 - \left(\frac{n_2\alpha_3}{n_3\alpha_2}\right)^2 > 0 \tag{47}$$

$$n_3^2\alpha_2^2 > n_2^2\alpha_3^2 \tag{48}$$

$$n_3^2\left(\frac{n_1^2}{n_2^2}\sin^2\theta - 1\right) > n_2^2\left(\frac{n_1^2}{n_3^2}\sin^2\theta - 1\right) \tag{49}$$

$$n_1^2\sin^2\theta\left(\frac{n_3^2}{n_2^2} - \frac{n_2^2}{n_3^2}\right) - \left(n_3^2 - n_2^2\right) > 0. \tag{50}$$

The left hand side of this relation is a minimum at the critical angle, where $\sin^2\theta = n_3^2/n_1^2$. We substitute this minimum value into (50) to yield the relation

$$\frac{n_1^2 n_3^2}{n_1^2}\left(\frac{n_3^2}{n_2^2} - \frac{n_2^2}{n_3^2}\right) - \left(n_3^2 - n_1^2\right) > 0 \tag{51}$$

$$n_3^4 > n_3^2 n_2^2 \tag{52}$$

which holds in the system of interest, since $n_3 > n_2$. The numerator can be proven positive by showing

$$\frac{n_2}{n_3\alpha_2}\frac{\partial\alpha_3}{\partial\theta} - \frac{n_2\alpha_3}{n_3\alpha_2^2}\frac{\partial\alpha_2}{\partial\theta} > 0 \tag{53}$$

$$\frac{n_1^2}{n_3^2}\frac{\sin\theta\cos\theta}{\left(\dfrac{n_1^2}{n_3^2}\sin^2\theta - 1\right)} - \frac{n_1^2}{n_2^2}\frac{\sin\theta\cos\theta}{\left(\dfrac{n_1^2}{n_2^2}\sin^2\theta - 1\right)} > 0 \tag{54}$$

and

$$\frac{n_1^2}{n_3^2}\frac{1}{\left(\dfrac{n_1^2}{n_3^2}\sin^2\theta - 1\right)} > \frac{n_1^2}{n_2^2}\frac{1}{\left(\dfrac{n_1^2}{n_2^2}\sin^2\theta - 1\right)} \tag{55}$$

$$\frac{n_1^2}{n_3^2}\frac{n_1^2}{n_2^2}\sin^2\theta - \frac{n_1^2}{n_3^2} > \frac{n_1^2}{n_2^2}\frac{n_1^2}{n_3^2}\sin^2\theta - \frac{n_1^2}{n_2^2} \tag{56}$$

$$n_3^2 > n_2^2 \tag{57}$$

which again holds true. D is positive for p-polarization.

For s-polarization, D is positive if

$$\frac{\partial\gamma_s}{\partial\theta} > 0$$

i.e.,

$$\frac{\dfrac{n_3}{n_2\alpha_2}\dfrac{\partial\alpha_3}{\partial\theta} - \dfrac{n_3\alpha_3}{n_2\alpha_2^2}\dfrac{\partial\alpha_2}{\partial\theta}}{1 - \left(\dfrac{n_3\alpha_3}{n_2\alpha_2}\right)^2} > 0. \tag{58}$$

We again split up the fraction and prove that the numerator and the denominator are both positive. For the denominator to be positive, it must satisfy

$$1 - \left(\frac{n_3\alpha_3}{n_2\alpha_2}\right)^2 > 0 \tag{59}$$

$$n_2^2\alpha_2^2 > n_3^2\alpha_3^2 \tag{60}$$

$$n_2\left(\frac{n_1^2}{n_2^2}\sin^2\theta - 1\right) > n_3\left(\frac{n_1^2}{n_3^2}\sin^2\theta - 1\right) \tag{61}$$

$$n_3^2 > n_2^2 \tag{62}$$

which is satisfied in our system. For the numerator to be positive, it must satisfy

$$\frac{n_3}{n_2\alpha_2}\frac{\partial\alpha_3}{\partial\theta} - \frac{n_3\alpha_3}{n_2\alpha_2^2}\frac{\partial\alpha_2}{\partial\theta} > 0 \tag{63}$$

$$\frac{1}{\alpha_3^2}\frac{\partial\alpha_3}{\partial\theta} - \frac{1}{\alpha_2^2}\frac{\partial\alpha_2}{\partial\theta} > 0 \tag{64}$$

$$\frac{n_1^2}{n_3^2}\frac{\sin\theta\cos\theta}{\left(\dfrac{n_1^2}{n_3^2}\sin^2\theta - 1\right)} - \frac{n_1^2}{n_2^2}\frac{\sin\theta\cos\theta}{\left(\dfrac{n_1^2}{n_2^2}\sin^2\theta - 1\right)} > 0. \tag{65}$$

This inequality was already proved in (54). Thus, terms B and D of (29) are positive for s and p-polarizations. This proves the inequality in equation (29) and the Goos–Hänchen time is always positive and greater in magnitude than the Gires–Tournois time.

IV. CONCLUSION

The time delay due to the Goos–Hänchen shift has been shown to be positive and greater in absolute magnitude than the Gires–Tournois time in a specific type of frustrated Gires–Tournois interferometer.

Unlike the Gires–Tournois time, the Goos–Hänchen time delay is accompanied by a spatial displacement of the beam. The effects of the spatial displacement and the Goos–Hänchen time delay result in a shift of the wavefronts of the reflected beam perpendicular to the wavevector of that beam relative to simple Fresnel reflection. Such a shift in the wavefronts does not produce a time delay measurable using interferometric techniques. As such, in this system, any interferometer will only be sensitive to the (negative) Gires–Tournois time delay, although the physical (2-D) time delay is, in fact, positive. Experiments are underway at the University of Toronto to measure the Gires–Tournois time delay using both classical and quantum [13], [3], [14] interferometric techniques.

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